

# TP-Model Transformation-Based-Control Design Frameworks



Péter Baranyi

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 Springer

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ISBN 978-3-319-19604-6                      ISBN 978-3-319-19605-3 (eBook)  
DOI 10.1007/978-3-319-19605-3

Library of Congress Control Number: 2016936784

Springer Cham Heidelberg New York Dordrecht London  
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# Preface

“*Conditā descrescit, vulgata scientia crescit.*”

My goal in this book is to share the benefits of TP model transformation-based solutions uncovered through work in my laboratory and to share some of our experiences in control design. I hope the frameworks introduced in the book will help to radically decrease the amount of analytical work that is performed, often unnecessarily, by researchers and engineers working in the field of control design optimization. If our experience can serve as any basis for generalization, many existing analytical approaches can be substituted by more flexible and effective numerical methods.

The TP model transformation-based frameworks provide a simple, generic, and flexible way to interface between identification stages and, primarily, linear matrix inequality-based control design theories. Further, they support stability verification purposes in general, even in cases where identification and design are based on very different representations. Finally, the presented frameworks lay the foundations for convex hull manipulation-based control design optimization.

I would like to express my appreciation to my friends Prof. Yeung Yam and Prof. Péter Várlaki for their strong support and for their help in shaping, through many discussions, a broader scientific and conceptual view behind the TP model transformation. I am indebted to the work of young researchers Dr. Béla Takarics, Dr. Péter Galambos, Dr. Ádám Csapó, Patricia Gróf, József Kuti, and Szöllösi Alexandra, who have helped in preparing a large number of experimental case studies and in extending the TP-tool MATLAB toolbox. I am grateful to Anna Szemereki for her help in managing all the related research work and projects that made it possible for the research group to focus on the research behind this book. Finally, I would like to thank our collaborators and graduate students, past and present, for their inputs and contributions to research on this subject.

Budapest, Hungary  
January 2016

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# Acronyms and Abbreviations

CHOSVD	Compact HOSVD
CNO	Close-to-normality
DoF	Degree of freedom
HOOI	Higher-order orthogonal iteration
HOSVD	High-order singular value decomposition
INO	Inverse normality
IRNO	INO and RNO
LMI	Linear matrix inequality
LMIs	Linear matrix inequalities
LPV	Linear parameter-varying
LTI	Linear time-invariant
NN	Nonnegativeness
NO	Normality
PDC	Parallel distributed compensation
qLPV	quasi-LPV
qNN	quasi-NN
qSN	quasi-SN
RHOSVD	Reduced HOSVD
RNO	Relaxed normality
SN	Sum normalization
SVD	Singular value decomposition
TP	Tensor product
TP model	Finite element TP-type polytopic model
TP <sup>+</sup>	Pseudo TP model transformation



# The Key Messages of the Book

The TP (tensor product) model transformation was originally proposed in [3, 12] and summarized in [15] for polytopic representation-based qLPV (quasi-linear parameter varying) control theories. The core of the TP model transformation was first introduced as an approach to the complexity reduction of fuzzy systems [13, 14, 74]. It transforms a function (which can be given via closed formulas or neural networks, fuzzy logic, etc.) into TP function form whenever such a transformation is possible. If an exact transformation is not possible, then the method determines a TP function that is an approximation of the given function. The TP model transformation also provides a trade-off between approximation accuracy and the complexity of the resulting TP function. These properties were investigated in [7, 15, 51, 69]. The HOSVD (higher-order singular value decomposition)-based canonical form of TP models was initiated in [10], and it was also proved that the TP model transformation is capable of numerically reconstructing this form [64]. A computationally relaxed variant of the TP model transformation was proposed in [8, 49]. A centralized variant of the transformation was given in [48]. Convex hull manipulation techniques were incorporated into the TP model transformation [4, 15, 71, 74].

Besides serving as a transformation of functions, however, the TP model transformation also represents a new concept in qLPV-based control. It is uniquely effective in manipulating the convex hull of polytopic forms and, as a result, has revealed and proved the fact that convex hull manipulation is a necessary and crucial step in achieving optimal solutions and decreasing conservativeness in modern LMI (linear matrix inequality)-based control theory [15, 66]. Hence, although the TP model transformation is just transformation (HOSVD of functions) from a mathematical point of view, it has nevertheless been successful in establishing a conceptually new direction in control theory and has laid the ground for further new approaches toward optimality.

Soon, it was extended to TP model transformation-based system control design framework for polytopic model and LMI-based system control through a series of

applications [4–6, 11, 52, 68]. A more complex design framework was proposed in [64]. A MATLAB toolbox was also created for this transformation and related functionalities; see [9].

Relying on the above properties, a variety of further control solutions have been proposed in the literature (prime examples can be found in [1, 2, 16, 18, 19, 21, 23, 24, 26–34, 38, 40, 42–45, 47, 53–55, 57, 59, 62, 63, 65, 70, 73, 75, 76]). Further applications of the TP model transformation in sliding-mode control were presented in [35–37, 39, 61, 67, 77–80].

Very recently published papers in the special issue “TP model transformation-based control system design” of the *Asian Journal of Control* are presenting further new directions [11, 17, 20, 22, 25, 41, 42, 46, 50, 56, 58, 60, 72].

In many respects, this book can be seen as a continuation of [15] that focuses primarily on the TP model transformation, with special attention to its capabilities related to approximation, complexity reduction, and convex hull manipulation. A few general thoughts were also given in that book on how these capabilities can be effectively applied in control design tasks. In the current book, our primary goal is to cover new aspects and frameworks of control design and optimization based on the TP model transformation and its various extensions. The chapters and the complex dynamical control tasks which they cover are organized so as to present and analyze the beneficial aspects of this family of approaches. Additionally, the book aims to convey the following messages to the reader:

- **Simple TP modeling**

The book demonstrates that the TP model transformation provides designers with a means of automatically transforming qLPV models (given by various different representations such as closed formulas or softcomputing techniques) to TP model form in a numerically tractable way. The TP form is a polytopic structure based on which LMI-based design approaches can be directly applied. Based on this capability, researchers and engineers are enabled to apply modern convex optimization-based approaches formulated in the form of LMIs to complex systems without the need to rely on complicated analytical derivations.

- **TP model manipulation-based possibilities for optimization**

Given that the TP model transformation provides an efficient method toward the manipulation of the convex hull defined by the vertices of the TP model, it brings to light the relatively unknown and rarely analyzed fact that LMIs (and the solvers too) in general are highly sensitive to the geometric properties of the convex hull, i.e., the number and the relative locations of the vertices. Based on this property, it is clear that besides LMI manipulation, convex hull manipulation aided by the TP model transformation can be an effective approach in the optimization of control performance. The book also points to the previously unknown fact that in searching for the optimal design of various components of a system (e.g., the controller and the observer), it is often desirable to apply different kinds of convex hull manipulations, and what is even more important, the simultaneous use of several TP model representations of a given model (i.e., based on different convex hulls) can lead to significantly improved control



performance. Thus, it may, for instance, be effective to design a controller based on a TP model representation that is built over a tight convex hull, while designing an observer based on a TP model representation that is built over a loose convex hull. The book presents design framework for this novel optimization possibility.

- **General framework for stability analysis**

One of the main messages of the book is that using the TP model transformation, it is possible to transform all components of a system to TP models with common weighting function structure. Based on this capability it becomes possible to perform LMI-based stability analysis on a wide range of systems. The primary criticism against soft-computing approaches in general is that no comprehensive framework exists that could be used to prove the stability of systems which were designed using a combination of heuristic techniques. Thus, for example, no comprehensive method exists which could be used to derive LMI-based stability verification to prove the stability of a system that was designed based on a combination of fuzzy and artificial neural network techniques (for instance, fuzzy controller and neural network observer are combined). Using the TP model transformation, however, it becomes possible to create a unified polytopic structure for all components of a systems; thus, LMI-based methods are directly applicable. As a result, a general stability verification method can be arrived at, which is capable of answering the criticisms cited earlier.

- **Standardized gateway between identification and design**

Based on the above, the TP model transformation provides a standardized interface between various heuristic identification tools used for different applications and the well-defined LMI-based design procedures. Thus, TP model transformation supports the conversion of any model to TP model form, irrespective of the identification technique which was used to create the model, and hence the validation of any model can proceed as if it was a TP model from the outset. The TP model transformation is capable of further manipulating the convex hull of the resulting models, as described above. Thus, TP model transformation can be used as an interface and preprocessing tool for further steps in LMI-based control design.

- **TP model-based design framework**

Given that the TP model transformation is well-suited to complexity reduction, convex hull manipulation, as well as the creation of unified TP model forms, it can be used to support the creation of complete control design frameworks. Within the frameworks, the entire design process can be effected on a reduced model (which results in reduced design complexity), and various TP model representations can be used for different system components (resulting in the advantages described earlier). As all steps of the framework are based on the TP model form, important simplifications can be made between neighboring steps, resulting in a compact and relaxed TP model-based control design method and framework. Complexity reduction in this case also allows for an increase in the effectiveness of LMI techniques.

- **Gateway to non-qLPV models and time-delayed systems**

The book demonstrates that the TP model transformation can be used to modify the interpretation of operational parameters such that, for instance, time-delay systems can be converted into qLPV systems (excluding delay) which considers time delay as an external system parameter. Based on this aspect, the applicability of well-established qLPV and LMI-based methods can be effectively extended to a wide range of problems involving time delay.

# Outline of the Book

The book is organized into five main parts. The first part (Part I: Chap. 2) proposes the generalized TP model transformation that includes various TP model manipulation techniques. Based on these manipulations, the second part (Part II: Chaps. 3–7) proposes control design frameworks with various beneficial features. The third part (Part III: Chaps. 8–12) of the book gives complex control design example focusing on one control design problem that have emerged very recently, in order to show how these frameworks can be applied to real-world problems and to study new features of design and effectiveness relevant to their use. The last two parts (Parts IV and V: Chaps. 13–18) focus on two real-world engineering control problems. The chapters are organized as follows:

Chapter 1 recalls some basic definitions and concepts about the TP model transformation presented in previous works. It recalls the definition of the TP function, as well as the TP model, which introduces the use of TP functions into the concept of polytopic qLPV models. The chapter also briefly discusses the conceptual similarities and differences between TP models and TS fuzzy models. This discussion makes clear that all model manipulation and LMI design concepts discussed in the book can be applied in fuzzy modeling and control design as well. The chapter also highlights the fact that the HOSVD-based canonical form of qLPV models can be extended. Thus, it redefines the previously published HOSVD-based canonical form.

Chapter 2 integrates various ideas about the TP model transformation into one conceptual framework and formulates the framework in terms of the generalized TP model manipulation. Several new extensions of the TP model transformation are proposed, such as the numerical reconstruction of quasi and “full,” compact and rank-reduced HOSVD-based canonical form of TP models and the bilinear-, multi-, pseudo-, and convex-TP model transformations. All of these extensions together form the generalized TP model transformation, which provides an effective tool to freely and readily manipulate the weighting functions and, hence, the convex hull defined by the vertexes of the TP models. Further, they provide a means to perform TP model-based main component analysis, as well as a host of complexity and accuracy trade-offs within TP models. All of these techniques form a generalized

tool for the convex hull manipulation specialized to TP models, which has a crucial role in polytopic model-based LMI design methods as will be discussed in the next part of the book. The chapter also gives some hints on how to use the TP model transformation to define and compute operations on and between TP models that provide further freedom toward TP model manipulation. Specifically, this chapter shows how to unify the weighting functions of preexisting TP models, i.e., how to find a common weighting function system to which all TP functions can be exactly transformed. The chapter also shows how to interpolate between pairs of TP models in such a way that their weighting functions are interpolated in a way that corresponds to a new TP model. This technique is very useful in the case of convex hull manipulation-based control performance optimization as will be shown in the context of design examples.

Part II of the book demonstrates that the proposed manipulation forms a new, effective, and necessary optimization step of polytopic models and LMI-based control design and that it can also be used to decrease conservativeness. This alleviates the problem that identification techniques are typically constructed based on the data and measurement set that is available and based on the type of system that is to be identified, irrespective of the kind of representation that is most suitable to the control design framework.

Chapters 3 and 4 demonstrate how the proposed generalized TP model transformation is unique in the sense that it bridges between various soft-computing-based identification techniques and the TP model. By association, the model makes it possible to merge soft-computing-based identification with polytopic model-based control design approaches. The chapter gives a discussion on the dual role of the TP model transformation as a final step of identification on the one hand and as a generalized “interface” on the other, which is capable of serving as a preprocessing step prior to the fulfillment of further design requirements (e.g., convex hull manipulation). This very unique feature of the TP model transformation makes it a very powerful tool, as it allows for the free combination of available design and identification techniques without the usual drawbacks of having to deal with incongruent mathematical derivations and representations. These advantages become even more clear when the extended TP model-based control design framework is applied to real-world problems.

Chapter 5 proposes the multi-TP model transformation-based stability verification framework, which is a tractable and non-heuristic framework that enables the stability verification of results obtained through (hybrid) soft-computing-based control design approaches, i.e., in which different system components can even be formulated in different representations (e.g., the functionality of a controller might be expressed through fuzzy rules, the observer might be formulated using a neural network, etc.). The multi-TP model transformation-based framework can provide an answer to the frequently emerging criticisms regarding the lack of mathematical stability verification techniques in soft-computing-based control design. The entire stability verification technique developed in the chapter can be fully automated and numerically executed in a reasonable amount of computing time.

Chapters 6 and 7 extends the TP model transformation-based design frameworks to non-qLPV systems and time-delayed systems.

In Part III Chap. 8 introduces the most recent version of the 3DoF nonlinear aeroelastic test apparatus (NATA) wing section model generated from real measurements. The model was presented and deeply elaborated in a series of papers published in the *Journal of Guidance, Control, and Dynamics*. This chapter takes the most recent model and extends it with Stribeck friction, a component which considerably increases the dynamic and modeling complexity of the system. The chapter then focuses on the problem of flutter suppression for the prototypical aeroelastic wing section. The flat plate airfoil is constrained to have 3DoF, i.e., plunge, pitch, and trailing-edge surface deflection. The main goal of the chapter is to describe and prepare a complex example for the next chapters, in order to study the effectiveness of various design techniques based on the TP model transformation.

Chapter 9 applies the TP model transformation in a very straightforward and “direct” way, i.e., without any convex hull manipulations, to the extended model of the aeroelastic wing section derived earlier. Different controllers are derived and the design also includes constraints on the control value and decay rate control. The chapter also provides an evaluation of the resulting controller based on numerical simulations. It is shown that the entire design process can be executed in an automated way with minimal human interaction

Chapter 10 continues further with the control design for the 3DoF aeroelastic model. The solution provided in the chapter incorporates convex hull manipulation. Through this aspect, the chapter demonstrates the little known fact that the use of different TP model representations of the system when designing the controller and observer can have beneficial effects on the resulting control performance. The chapter shows that by tightening the convex hull of the TP model, the conservativeness of the control design is decreased; however, by loosening the convex hull, the observer performance is improved. Hence, the chapter suggests that the trade-off between these contradictory requirements can be optimized through convex hull manipulation so that the performance of the entire system as a whole is optimal. A further takeaway is that if no compromise is accepted between the requirements, defining separate convex hulls for the controller and observer (tight and loose, respectively), further improvements can be achieved. The chapter shows that the TP model transformation can be used to achieve these objectives. Discussions in the chapter are supported by numerical simulation-based evaluations.

Chapters 11 and 12 evaluates the effectiveness of the convex hull manipulation in control design. This manipulation includes the shape and the complexity of the convex hull. As in previous chapters, the analysis is applied to the 3DoF aeroelastic wing section.

Part IV presents a complex example of the control design of the dual-excenter vibration actuator. Vibration actuators are widely used, for instance, in handheld devices to provide vibrotactile feedback or silent notification to users. In most cases, miniature DC motors with an eccentric rotor or the so-called coin-type shaftless vibration motors are utilized. The common disadvantage of the single rotor design is that the frequency and the intensity of the generated vibration cannot be

adjusted separately. On the other hand, the construction that is composed of two independently driven coaxial eccentric rotors—which makes for a strongly coupled nonlinear system—allows for the separate control of the frequency and amplitude by the adjustment of the angular speed and the total eccentricity. The chapter presents a complete control design approach based on qLPV modeling and LMI-based synthesis utilizing the TP model transformation to determine the convex polytopic representation of the parameter-dependent nonlinear system. The design approach is demonstrated via a concrete numerical example using the parameters of a real dual-excenter prototype device. The control performance is validated through numerical simulations. This case study goes through a complete nonlinear control problem from the modeling phase to the design of the implementation-ready controller while drawing generalizable lessons on TP model transformation-based control design.

Part V presents an example that focuses on impedance model control applied to a force feedback telemanipulation system in which time delay leads to significant problems. Through the example, the chapter studies the use and effectiveness of the TP<sup>r</sup> model transformation. The chapter introduces a widely applied impedance control scheme for force reflecting telemanipulation and then focuses attention on the theoretical limitations of the stability of such systems under time delay. Stability is then analyzed in terms of the maximum time delay under which the impedance model still remains stable. Finally a stabilizing controller is designed.

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