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# Astrophysical Black Holes

 Springer

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# Preface

The first black hole solution of Einstein's field equations was discovered by Karl Schwarzschild. It represents the exact solution for the metric tensor of a point mass in an otherwise empty space, and was later recognised to be a limiting case of a more general solution found by Roy Kerr. The black hole solution hides at its center a mathematical singularity that cannot be eliminated by any change of coordinates. There, curvature scalars that are invariant under coordinate transformations diverge to infinity. A black hole is indeed one of the simplest (mass, spin and electric charge fully define its properties) and yet most mysterious concepts conceived by the human mind. Black holes became an iconic figure in popular culture. The recent Hollywood science fiction movie *Interstellar* features a supermassive Kerr black hole as the uncredited leading actor.

Among the many questions posed by the theory of general relativity, one was particularly intriguing: are black holes real, observable (albeit indirectly) astrophysical objects, or we should consider them as a mathematical concept? The idea that black holes do indeed form in nature developed as soon as it was recognised that stars cannot remain in stable equilibrium when the pressure support against gravity drops to the point that the total energy of the star is no longer a minimum. Loss of dynamical stability occurs under a variety of conditions: when the star is either supported by the pressure of degenerate relativistic electrons or neutrons in cold, dense matter, or by radiation pressure in a hot, tenuous medium. It occurs further when energy from nuclear reactions is deposited, triggered by the ensuing instability. In addition, the instability is seeded in any star and regardless the equation of state of matter when the non-linear nature of the gravitational interaction becomes important, in the strong field limit.

While the concept of a black hole as the natural endpoint of the evolution of a massive star was accepted, a more exotic flavour of these objects was envisaged. The suggestion of the existence of supermassive black holes, weighting from millions to billions solar masses, originated in the early 1960 following the discovery of the first quasars. Quasars are active galactic nuclei that are so luminous that often outshine their own host galaxies. Their radiation is emitted across a very broad spectrum, from the X-rays to the far-infrared, and in a fraction of cases, from TeV energies to

radio waves. Variability on short timescales soon revealed that the emitting region is only a few light years across. All observational evidences made clear that a stellar origin of the emission was highly unpalatable: the hypothesis that a supermassive black hole, lurking at the center of some galaxies, was responsible of the quasar phenomenology was then soon prompted.

Now we know that supermassive black holes are a common presence in the center of galaxies, and are a key ingredient in galaxy's evolution. Indeed, over the last 15 years, thanks to the unprecedentedly high angular resolution and sensitivity of astronomical observing facilities, it has been possible to measure the spatial distribution and spectroscopic velocities of stars not only in our Galactic Center, but also in the nuclei of several nearby galaxies. All observations invariably point towards the presence of a massive dark object with inferred masses similar to those thought to power quasars. The concept that (almost) every spheroid harbours a supermassive black hole in its very center is one of the greatest discovery in modern astrophysics.

This volume treats what we called "astrophysical black holes", i.e. black holes seen in and from an astrophysical perspective. The field is vast but, in large degree, fairly well defined. Based on their lectures given at the 2012 SIGRAV school, organised by the Italian Society of Relativity and Gravitation for an international group of Ph.D. students and supported by the Università degli Studi dell'Insubria and by the Italian Institute of Nuclear Physics (INFN), leading renowned experts give here their own view from many different angles of the complex black hole phenomenon, highlighting the basic principles and observations, and the challenges ahead in one of the most fascinating field in contemporary astrophysics and cosmology.

Luciano Rezzolla opens this volume with a detailed review of the physics of gravitational collapse in GR. He first illustrates the simplest and yet revealing model of gravitational collapse: the Oppenheimer-Snyder collapse of a dust sphere to a black hole, paying special attention to the dynamics of trapped surfaces, such as apparent and event horizons. The contribution then turns to the more realistic case of the gravitational collapse of a self-gravitating fluid sphere, exploiting all the insight gained with dust, and introducing the very idea of black hole. Rezzolla then discusses how we can learn about the properties of black holes, spherical and axisymmetric, considering the motion of test particles. The second part of this first chapter is instead dedicated to a rather different route leading to the formation of an isolated black hole: the merger of a binary of black holes. In particular, it will be shown how it is possible to compute the mass and spin of the final black hole simply in terms of an algebraic expression containing information on the properties of the two initial black holes. Rezzolla contribution is fully founded on GR, still it remains essentially devoted to researchers with astrophysical interests.

Chris Nixon and Andrew King bring then our attention to some of the latest development in accretion disks theory, namely the physics of warped disks. The authors show how any non-axisymmetric force, a general condition in astrophysics, inevitably led to the formation and propagation of warps in gas orbiting compact objects, specifically black holes. In the chapters the authors discuss the two types

of warp propagation, through waves and diffusion. They derive the evolution equations and discuss their interpretation. The contribution continues with a detailed description of the viscosity, and in particular of the relation between the small scale turbulent viscosity and its role in shaping the effective viscosities which control the dynamics of warped discs. Finally Nixon and King discuss some major results and some outstanding problems in understanding this complex and subtle accretion disc behaviour.

The book continues with the contribution of Rob Fender and Teo Muñoz-Darias, focused on the population of stellar-mass black holes in our and other galaxies. In particular the authors focus on how we can attempt to balance the available accretion energy with feedback to the environment via radiation, jets and winds, considering also possible contributions to the energy balance from black hole spin and advection. Fender and Muñoz-Darias review the methods which are used to estimate these quantities, and once these methods have been outlined, they work through an outburst of a black hole X-ray binary system, estimating the flow of mass and energy through the different accretion rates and states. While the focus is on feedback from stellar mass black holes in X-ray binary systems, the contribution considers also the applicability of what we have learned to supermassive black holes in active galactic nuclei. Finally, the two authors review the coupling between accretion and feedback in neutron stars, and show that it is very similar to that observed in black holes, which strongly constrains how much of the astrophysics of feedback can be unique to black holes.

With Andrea Merloni we then move to the realm of massive, and super-massive, black holes. In his essay, Merloni reviews the current state of affairs regarding the study of the evolution of the black hole population in the nuclei of galaxies. He first describes the observational techniques used to survey the sky in search of signs of black holes activity, and the progresses made on constraining the phenomenological appearance of AGN. The chapter is then devoted to the physical description of the processes thought to be responsible for the observed energy emission in luminous AGN, focusing in particular on the properties of AGN accretion discs, coronae and the IR-emitting dusty clouds. The contribution continues with a concise overview of the current state of the art of AGN luminosity function studies at various wavelength, encapsulating our knowledge about the overall population cosmic evolution. The final part of the chapter is devoted to a general discussion of the methods by which we use the evolutionary study of the AGN population to infer additional global physical properties of the process of accretion onto and energy release by supermassive black holes.

The volume continues with David Merritt, who deals with the study of orbital motion in galactic nuclei. The author shows how encounters between stars and stellar remnants at the centers of galaxies drive many important processes. The fact that these encounters take place near a supermassive black hole alters the dynamics in a number of ways. As an example, the orbital motion is quasi-Keplerian so that correlations are maintained for much longer than in purely random encounters; moreover, relativity affects the motion, through mechanisms like precession of the periastron and frame dragging, and the black hole spin is affected, directly by

capture and indirectly by spin-orbit torques. Merritt describes the interplay between these processes, showing that GR can be crucially important even at distances that are thousands of gravitational radii from the event horizon.

The essay of Michela Mapelli and Alessia Gualadris continues in treating the dynamics around massive black holes, focusing now to our own Galactic Center, as one of the most studied and yet enigmatic places in the Universe. The authors show how the Galactic center is the ideal environment to study the extreme processes that take place in the vicinity of a supermassive black hole, with a detailed review of the main scenarios proposed to explain the formation and the dynamical evolution of the early-type stars in such hostile environment. In particular, the most popular *in situ* scenarios (accretion disc fragmentation and molecular cloud disruption) and migration scenarios (star cluster inspiral and Hills mechanism) are discussed. The authors' focus is finally given on the most pressing challenges that must be faced to shed light on the process of star formation in the vicinity of a super-massive black hole.

The final chapter of this volume contains the contribution of Thibault Damour and Alessandro Nagar. The authors give a comprehensive description of the two-body problem in GR. After reviewing some of the methods used to tackle this problem (and, more generally, the N-body problem), the authors focus on a new approach to the motion and radiation of binary systems, called the Effective One Body (EOB) formalism, reviewing the basic elements of this formalism, and recent developments. Amour and Nagar then show the EOB formalism is able to provide accurate descriptions of the dynamics and radiation of various binary systems (comprising black holes or neutron stars) in regimes that are inaccessible to other analytical approaches. In synergy with other tools and methods, the EOB formalism is shown to be a promising way of computing the very many accurate template waveforms that are needed for gravitational wave data analysis purposes.

Como, Italy

Francesco Haardt



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