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Random Walks, Random Fields, and Disordered Systems

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Foreword

These Lecture Notes contain detailed expositions of five (out of six total) lecture series delivered at the 2013 Prague School on Mathematical Statistical Mechanics. The lecture series aimed to address some of the currently most rapidly developing subjects at the interface of probability theory, statistical physics, combinatorics, and computer science. One common thread running through all lectures is the prominent role played by random walks and random fields; these appear both as primary objects of study or as tools to represent other quantities of interest. Another important common aspect is the role of underlying disorder that is responsible for many spectacular effects found in the behavior of these systems. We will now describe the content of the forthcoming chapters in more detail, highlighting the important features and further connections.

The opening chapter, contributed by Anton Bovier, summarizes recent advances in the understanding of the Branching Brownian Motion. This is a process of particles that perform independent Brownian motions until they branch, at random times, to form more Brownian particles that henceforth move (and branch) independently. Such models are naturally interesting for population dynamics, although other uses can be found in the literature as well. The present notes address the Branching Brownian Motion from the perspective of statistical mechanics of disordered systems, particularly, spin glasses with Gaussian disorder. What connects these together is the reliance on extreme order statistics, a method for systematic study of the maximum and, in fact, all near-maximal values of a large number of random variables. There is also a close link to the subject of random fields as the extrema of the Branching Brownian Motion turn out to behave quite similarly to those of the two-dimensional Gaussian Free Field. The latter is a topic that plays an important role in several other chapters of the Lecture Notes as well.

The second chapter, written by David Brydges, explains how the behavior of the four-dimensional weakly self-avoiding walk can be studied using the methods of field theory and renormalization group. A representation of a weakly self-avoiding random walk by means of an interacting random field (specifically, the ϕ^4-theory) goes back several decades, but it was only recently that this connection could be exploited to obtain rigorous conclusions about the random walks as well. This is particularly challenging in four spatial dimensions, where the renormalization group has to deal with (what are called) marginal terms. As a result, logarithmic corrections appear next to the ordinary "diffusive" or "free-field like" scaling of the two-point correlation function known to occur above the upper critical dimension. Brydges's notes explain all necessary background for entering this field and, in particular, his own extensive papers on the subject that have been circulated recently.

The third contribution to these Lecture Notes, written by Amin Coja-Oglan, takes us to the world of discrete structures exhibiting phase transitions as the parameters of the problem vary. The "interaction" in these systems is usually of a rather simple kind; what makes the problem hard is the disorder that is built into the system. The main focus of the notes is a specific tool, called the cavity method, whose name fittingly describes the basic idea: evaluate the change of the physical properties of the system when a "cavity" is created by removing a single constituent. The cavity method is commonly used by physicists to study spin-glass systems; unfortunately, little of it has been put on rigorous footing until recently. Coja-Oghlan's notes describe some of the mathematical advances using salient examples of the Random-Graph Coloring and the Random Satisfiability Problem. Various other concepts that arise in this subject area are also explained and used in practice, e.g., the replica symmetry breaking, belief propagation, or quiet planting.

The fourth chapter of these notes, contributed by Dmitry Ioffe, is a survey of recent advances in the area of polymer models with disorder. The focus is on the ballistic regime, i.e., the situation when the polymer spreads linearly in space. The polymer itself is subject to self-interactions as well as interactions with a random environment. A rigorous version of the Ornstein-Zernike theory is reviewed, which gives full control of the limiting distribution of the polymer endpoint in the ballisticity regime. The role of a strong disorder in low spatial dimensions and a weak disorder in high spatial dimensions is discussed. Renormalization methods are employed to make a link to effective models that are easier to study.

The final chapter of the notes, written jointly by Greg Lawler and Jacob Perlman, summarizes Lawler's lectures on random walk loop-soup models. Here, a loop-soup is a family of random closed paths on a graph or in the Euclidean space that are subject to an interaction that depends on (what Lawler and Perlman call) an acceptable weight. Several two-dimensional systems of interest have a natural representation by way of a loop-soup measure, the self-avoiding walk being one natural example. The notes give an introduction to the approach based on complex weights and the relation between loop soups, the loop-erased random walk and uniform spanning trees. There are also natural connections to the Gaussian Free Field as well as other topics arising elsewhere in these Lecture Notes.

These Lecture Notes are primarily aimed at early graduate students with only a modest background in probability and mathematical physics. Notwithstanding, the notes will be enjoyed also by seasoned researchers as well as general audiences interested in learning about recent advances in the above fields.

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