

Editors-in-Chief:

J.-M. Morel, Cachan

B. Teissier, Paris

Advisory Board:

Camillo De Lellis, Zurich

Mario di Bernardo, Bristol

Alessio Figalli, Austin

Davar Khoshnevisan, Salt Lake City

Ioannis Kontoyiannis, Athens

Gabor Lugosi, Barcelona

Mark Podolskij, Aarhus

Sylvia Serfaty, Paris and NY

Catharina Stroppel, Bonn

Anna Wienhard, Heidelberg

More information about this series at

<http://www.springer.com/series/304>



FONDAZIONE
CIME
ROBERTO CONTI

CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

Fondazione C.I.M.E., Firenze

C.I.M.E. stands for *Centro Internazionale Matematico Estivo*, that is, International Mathematical Summer Centre. Conceived in the early fifties, it was born in 1954 in Florence, Italy, and welcomed by the world mathematical community: it continues successfully, year for year, to this day.

Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

C.I.M.E. Director	C.I.M.E. Secretary
Elvira MASCOLO	Paolo SALANI
Dipartimento di Matematica "U. Dini"	Dipartimento di Matematica "U. Dini"
Università di Firenze	Università di Firenze
viale G.B. Morgagni 67/A	viale G.B. Morgagni 67/A
50134 Florence	50134 Florence
Italy	Italy
e-mail: mascolo@math.unifi.it	e-mail: salani@math.unifi.it

For more information see CIME's homepage: <http://www.cime.unifi.it>

CIME activity is carried out with the collaboration and financial support of:

- INdAM (Istituto Nazionale di Alta Matematica)
- MIUR (Ministero dell'Università e della Ricerca)

Housseem Haddar • Ralf Hiptmair • Peter Monk •
Rodolfo Rodríguez

Computational Electromagnetism

Cetraro, Italy 2014

Alfredo Bermúdez de Castro, Alberto Valli
Editors



Authors

Houssein Haddar
INRIA and CMAP, Ecole Polytechnique
Palaiseau, France

Ralf Hiptmair
Department of Mathematics
ETH Zürich
Zürich, Switzerland

Peter Monk
Department of Mathematical Sciences
University of Delaware
Newark, DE, USA

Rodolfo Rodríguez
CI²MA and Departamento de Ingeniería
Matemática
Universidad de Concepción
Concepción, Chile

Editors

Alfredo Bermúdez de Castro
Departamento de Matemática Aplicada
Universidade de Santiago de Compostela
Santiago de Compostela, Spain

Alberto Valli
Dipartimento di Matematica
Università di Trento
Povo (Trento), Italy

ISSN 0075-8434

ISSN 1617-9692 (electronic)

Lecture Notes in Mathematics

ISBN 978-3-319-19305-2

ISBN 978-3-319-19306-9 (eBook)

DOI 10.1007/978-3-319-19306-9

Library of Congress Control Number: 2015944330

Mathematics Subject Classification (2010): 65M60, 65M38, 35Q61, 65M32, 65Z05

Springer Cham Heidelberg New York Dordrecht London

© Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

Preface

The CIME School on “Computational Electromagnetism” was held in Cetraro (Italy) from June 9 to June 14, 2014.

Numerical approximation of partial differential equations in electromagnetism has attracted more and more attention over the last few decades. On the one hand, new theoretical results about the solutions of Maxwell’s equations have been obtained, highlighting some properties useful for more efficient discretizations; on the other hand, innovative numerical schemes have been proposed and analyzed, based on finite elements, boundary elements or finite differences.

In particular, the use of finite elements saw a fundamental change of paradigm about 30 years ago, when it became clear that finite elements with degrees of freedom expressed in terms of edge and face integrals were the most suited for numerical approximation in electromagnetism.

These notes are based on the four courses delivered in Cetraro, and the aim is to present some recent and significative results related to different aspects of numerical simulation of real-life electromagnetic problems, including some more theoretical results that are useful in devising and analyzing the approximation algorithms.

The content of the courses is the following.

Ralf Hiptmair’s contribution is concerned with the spatial discretization of Maxwell’s equations in a bounded domain by means of edge elements. The presentation covers different aspects: the modeling of electromagnetic problems from the point of view of exterior calculus, starting from the classical Maxwell’s equations and arriving to their variational formulation; the introduction of finite element exterior calculus and the construction of discrete differential forms, with the aim of devising suitable Galerkin discretizations of Maxwell’s equations; the numerical analysis of the discretized equations in order to establish a priori discretization error estimates, with special emphasis on the Maxwell cavity problem.

The chapter written by Rodolfo Rodríguez deals with the eddy current model in harmonic regime and its numerical approximation by finite element methods. Several formulations are presented: in the first one the unknowns are the magnetic field in conductors and a scalar magnetic potential in dielectrics, with current sources as boundary data; the second one is a saddle-point mixed formulation based

on the magnetic field in the whole domain, and on the electric field in dielectrics; the third one is expressed in terms of a scalar magnetic potential in a part of the dielectric region and of a vector magnetic potential in the complementary part. The stability and convergence properties of the associated finite element schemes are presented.

Peter Monk (together with Jieli Li and Daniel Weile) gives a thorough description of the time domain integral equation method, a major tool in the computational analysis of electromagnetic scattering problems. The presentation is mainly concentrated on the Electric Field Integral Equation, and the issues of convergence and stability are considered in detail, for both spatial and temporal discretization; in particular, an in-depth analysis of Convolution Quadrature techniques for time discretization is presented. As a numerical example, the scattering of electromagnetic waves from perfectly conducting objects is described. The problem of scattering by a homogeneous penetrable body is also analyzed, leading to the introduction of a new boundary integral operator related to the magnetic field.

The final contribution, due to Housseem Haddar, is an overview of the so-called “qualitative” methods for inverse electromagnetic scattering problems. The focus is on the inverse geometrical problem, namely, on recovering the shape of some inclusion from the measurement of scattered electromagnetic waves. An analysis of the linear sampling method, originally proposed by Colton and Kirsch in 1996, and of some more recent variants of it is presented in detail for the full three-dimensional electromagnetic problem. An additional related topic is also addressed: the problem of the existence of transmission eigenvalues (values of the frequency for which the interior transmission problem is not well posed).

It was a pleasure to work with all the lecturers. We would like to thank them for their efforts in presenting these important topics in a clear yet detailed way, and for their contribution to the nice atmosphere that characterized the entire school.

Finally, we would like to thank CIME for having accepted our proposal, for the efficient organization and for having hosted the school in a beautiful location. Special thanks are addressed to GNCS-INdAM and to the Departamento de Matemática Aplicada of the University of Santiago de Compostela for their financial support.

Santiago de Compostela, Spain
Povo (Trento), Italy
March 2015

Alfredo Bermúdez de Castro
Alberto Valli

Contents

Maxwell's Equations: Continuous and Discrete	1
Ralf Hiptmair	
Numerical Approximation of Maxwell Equations in Low-Frequency Regime	59
Rodolfo Rodríguez	
Time Domain Integral Equation Methods in Computational Electromagnetism	111
Jielin Li, Peter Monk, and Daniel Weile	
Analysis of Some Qualitative Methods for Inverse Electromagnetic Scattering Problems	191
Houssem Haddar	