Lecture Notes in Physics

Volume 907

The Lecture Notes in Physics

The series Lecture Notes in Physics (LNP), founded in 1969, reports new developments in physics research and teaching-quickly and informally, but with a high quality and the explicit aim to summarize and communicate current knowledge in an accessible way. Books published in this series are conceived as bridging material between advanced graduate textbooks and the forefront of research and to serve three purposes:

- to be a compact and modern up-to-date source of reference on a well-defined topic
- to serve as an accessible introduction to the field to postgraduate students and nonspecialist researchers from related areas
- to be a source of advanced teaching material for specialized seminars, courses and schools

Both monographs and multi-author volumes will be considered for publication. Edited volumes should, however, consist of a very limited number of contributions only. Proceedings will not be considered for LNP.

Volumes published in LNP are disseminated both in print and in electronic formats, the electronic archive being available at springerlink.com. The series content is indexed, abstracted and referenced by many abstracting and information services, bibliographic networks, subscription agencies, library networks, and consortia.

Proposals should be sent to a member of the Editorial Board, or directly to the managing editor at Springer:

Christian Caron
Springer Heidelberg
Physics Editorial Department I
Tiergartenstrasse 17
69121 Heidelberg/Germany
christian.caron@springer.com

More information about this series at http://www.springer.com/series/5304

Valerio Faraoni

Cosmological and Black Hole Apparent Horizons

Springer

Valerio Faraoni
Physics Department
Bishop's University
Sherbrooke, QC, Canada

ISSN 0075-8450 ISSN 1616-6361 (electronic)
Lecture Notes in Physics
ISBN 978-3-319-19239-0 ISBN 978-3-319-19240-6 (eBook)
DOI 10.1007/978-3-319-19240-6

Library of Congress Control Number: 2015943318

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

To my sisters Adriana and Lilly

Preface

Currently, substantial research efforts are devoted to understanding the physics of horizons. A horizon is a surface which separates a region of space-time which is accessible to an observer from one which is not and from which this observer cannot receive light or other physical signals. This feature gives rise to interesting physics: with the emphasis given in modern times to the role played by information in theoretical physics, it is easy to guess that a horizon will produce interesting physical phenomena. If entropy is understood as information entropy, then a horizon which hides information should be attributed some entropy. This is in fact what the black hole thermodynamics developed in the 1970s found. The discovery of Hawking radiation from black hole horizons made possible the development of black hole thermodynamics, a remarkable and beautiful construct which shows that, indeed, there is very interesting physics associated with horizons. Already in special relativity without gravity, uniformly accelerated observers experience acceleration horizons. When gravity is introduced, one encounters black hole and cosmological horizons. Then, studying black holes, one meets inner, outer, Cauchy, and extremal horizons, and in cosmology there are particle, event, de Sitter, and apparent horizons.

The pioneers who developed black hole mechanics and thermodynamics in the 1970s discussed *stationary* black holes and *event* horizons. Dynamical situations such as gravitational collapse, black hole evaporation by Hawking radiation, and black holes interacting with nontrivial environments and exchanging mass-energy require that the concept of event horizon be generalized. Conceivable dynamical situations include black holes accreting surrounding fluids, black holes immersed in a cosmological background, and, most significantly, black holes emitting (and possibly absorbing) Hawking radiation, which becomes significant in the last evolutionary stages. If a black hole is placed in a nontrivial environment, its mass-energy should be also the internal energy which we need to account for in the first law of thermodynamics. This mass-energy must be defined carefully, usually with some quasi-local notion, which in turn is sometimes related to the notion of *apparent* horizon.

In dynamical situations, it is not clear what is meant by "black hole" because the most salient feature of a black hole is precisely its horizon, and the event horizon familiar from stationary black holes turns out to be essentially useless for practical purposes in dynamical space-times. This major obstacle appears because the definition of event horizon requires the knowledge of the entire future of space-time, which is physically impossible to achieve in nonstationary situations. The ambiguity in the notion of "horizon" therefore implies murkiness in the concept of "black hole" itself. Simultaneously, thanks to the increase in the power of modern supercomputers, great theoretical efforts are now made to predict in detail the waveforms of gravitational waves emitted by black holes. These waveforms are needed to build banks of templates to separate signals from noise in the laser interferometric detectors of gravitational waves. The notion of event horizon is of little use in the numerical study of fast astrophysical processes producing those gravitational waves. Instead, "black holes" are routinely identified with outermost marginally trapped surfaces and apparent horizons in numerical research. Hence, a part of the research community is still focused on event horizons, while another part dismisses it altogether and uses horizon surfaces, the role of which is not yet understood clearly. This dichotomy needs to be addressed, and this work is intended to give a contribution in this direction.

This book contains a series of graduate-level lectures introducing the main problems in this area of theoretical physics. The first three chapters are pedagogical in nature, while the remaining two report a series of "case studies" to which the concepts of apparent and trapping horizon are applied. They consist of relatively rare analytic solutions of Einstein's theory and of scalar-tensor and $f(\mathscr{R})$ gravity which appeared in the literature and contain, at least in certain space-time regions, black hole and cosmological apparent horizons. The dynamics of apparent horizons can be rather bizarre and reserves several surprises. The phenomenology of apparent horizons known thus far is described and analyzed. While this field of research is definitely not settled and the last word is not said on any of the issues examined, these lectures aim at collecting and summarizing the existing results and providing an introduction and a toolkit for researchers approaching this field, especially graduate students. An extensive bibliography refers the reader to specific points which cannot be discussed in a single volume. I hope that these lectures will be stimulating and that some of my readers will soon find new directions for this area of research.

Sherbrooke, Canada
April 2015

Valerio Faraoni

Acknowledgments

I am indebted to many people for their contributions to the material discussed in this book, and my thanks go first of all to my collaborators and to my students: Thomas Sotiriou, Vincenzo Vitagliano, Stefano Liberati, Angus Prain, Alex Nielsen, Audrey Jacques, Andres Zambrano, and Roshina Nandra.

For discussions or comments, or for the occasional e-mail pointing out references that I missed, I am grateful to Hideki Maeda, Matt Visser, Daniel Guariento, John Barrow, Eric Poisson, Ivan Booth, Luciano Vanzo, Sergio Zerbini, Rituparno Goswami, Bibhas Majhi, Andrzej Krasiński, Kayll Lake, Viqar Husain, Timothy Clifton, Sergei Odintsov, Emilio Elizalde, Sebastiano Sonego, Nemanja Kaloper, Bahram Mashhoon, Salvatore Capozziello, Valeri Frolov, Sarah Shandera, Carlos Herdeiro, José Senovilla, and Rong-Gen Cai. Many thanks to Angus Prain for drawing the figures and to Aldo Rampioni and Kirsten Theunissen of Springer for their support and friendly assistance during the writing of this book.

This research is supported by Bishop's University and by the Natural Sciences and Engineering Research Council of Canada.

Contents

Symbols and Acronyms

g_{ab}	metric tensor
g^{ab}	inverse metric tensor
g	metric determinant
η_{ab}	Minkowski metric
∇_a	covariant derivative operator
Γ^a_{bc}	Christoffel symbols
R_{ab}	Ricci tensor
$\mathscr{R} = R^a{}_a$	Ricci scalar
$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R$	Einstein tensor
T_{ab}	matter energy-momentum tensor
Λ	cosmological constant
l_{Pl}	Planck length
k^a, ξ^a	Killing vectors
K^a	Kodama vector
κ	surface gravity
τ	Kodama time
a	uniform acceleration of Rindler observers
$d\Omega^2_{(2)} = d\theta^2 + \sin^2\theta\, d\varphi^2$	line element on the unit 2-sphere
R	areal radius
$\bar{r}$	isotropic radius
η	conformal time
η^a	geodesic deviation vector
θ_l	expansion of a null geodesic congruence with tangent field l^a
M_{MSH}	Misner-Sharp-Hernandez mass
k_B	Boltzmann constant
T	(absolute) temperature
S	entropy
$\mathscr{A}$	horizon area

k	curvature index of FLRW space
a	scale factor of the FLRW metric
$H \equiv \dot{a}/a$	Hubble parameter of FLRW space
$q \equiv -\ddot{a}a/\dot{a}^2$	deceleration parameter of FLRW space
$\dot{f}$	derivative of f with respect to comoving time
$\mathscr{I}^+$	future null infinity
$J^-\left(\mathscr{I}^+\right)$	causal past of future null infinity
$\equiv$	equal by definition
$\doteq$	equality valid in General Relativity in FLRW space with a perfect fluid as the matter source

FLRW	Friedmann-Lemaître-Robertson-Walker
AH	apparent horizon
TH	trapping horizon
EH	event horizon
PH	particle horizon
FOTH	future outer trapping horizon
FITH	future inner trapping horizon
PITH	past inner trapping horizon
MOTS	marginally outer trapped surface
MTT	marginally trapped tube
MOTT	marginally outer trapped tube
iff	if and only if