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Marshall–Olkin Distributions - Advances in Theory and Applications

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Preface

Systemic risk is a key problem of this century. One of the most interesting methodological solutions to this problem was given in the past century in the work of Albert Marshall and Ingram Olkin. This book collects advances in this theory presented at the international conference opening the academic year of the Graduate Course in Quantitative Finance at the University of Bologna, held in Bologna in October 2013.

A systemic event is something that affects a set of objects at the same time. With society and the economy becoming global and with unregulated industrial development leading to more extreme kinds of risk, the relevance of systemic risk has dramatically increased. Air pollution and dangerous industrial waste represent a common factor affecting life expectancy of individuals, particularly in specific geographic regions or clusters of population in less developing countries. By the same token, the developments in medical sciences provide a common factor responsible for longer expected lives, particularly in the developed world. Moreover, all of this translates into common risk factors for the insurance sectors, for financial intermediaries, for firms and the society as a whole.

For these reasons the celebrated Marshall–Olkin model, published in 1967, is one of the best tools to address the analysis of risk in this century, the concept of risk being understood in its widest meaning: from that of stopping a machine to that of ending human lives, from natural catastrophes destroying everything built on a region of land to financial catastrophes triggering the default of clusters of firms or banks.

The kernel of the idea is very simple. There is an event that may kill me, one that may kill you, and one that could kill both of us at the same time. Even this raw idea, without any more structure, raises important questions, taking us beyond the technicalities of the model. The only fact that both you and me are exposed to a common shock induces dependence between our lives, because there are scenarios with positive probability in which we die together. This is a special kind of dependence. It is dependence that does not depend on any of us. It is a kind of dependence of which neither you or I carry the blame. It is a sort of background dependence that links our lives because we are located in the same region, we breathe the same air, we work in the same firm, fly on the same airplane. Our lives are dependent because we share the same exposure to some act of the Diabolic Mrs. Nature. For instance, we are exposed to the same catastrophe that may occur in our common region; we are exposed to the same disease carried by the infectious air we are breathing around the same chimney; we are exposed to lay-off if the firm in which we work closes down; we are exposed to die at the same time if the airplane in which we both travel is going to crash. Therefore, systemic risk is all about a background risk changing while we move in space and time, but that in general has moved forward to the front of the scene in the current century.

The structure imposed on the original Marshall–Olkin model was as soft as possible, and this would highlight even more this kind of irresponsible dependence. The shocks killing individuals and that killing all of them were assumed to be independent. They were all assumed to be generated by processes with lack of memory, which are processes for which the past does not have any impact on the lifetime expected in the future. Marshall and Olkin found that in their model this property would carry over to the elements of the cluster. For each of them the intensity, that is, the instantaneous probability of event occurrence to one element in the cluster, would be simply decomposed as the sum of the intensity of the systemic event and that of the idiosyncratic event. While the invariance of the lack of memory property is an important element of the model, which has raised curiosity and discussion among mathematicians, the idea of decomposition of the intensity is also prone to just the opposite need: the possibility to model ageing effects in a flexible way. In fact, invariance of the intensity of life ending is not common in nature, and it would be better to say that it is more the exception than the rule. However, even from the point of view of ageing, the linear decomposition allows an important range of flexibility. For example, in life insurance, the probability of death of an individual could be modelled by limiting ageing to the idiosyncratic part of intensity while keeping invariant the systemic intensity part, or even allowing for a reverse ageing effect common to all individuals and due to the innovation process in medical and healthcare sciences.

It is well known that the best way to provide flexibility to a multivariate model is to extract the dependence function, also known as the copula function, so leaving full flexibility to design the marginal distributions. Even from this point of view, the Marshall–Olkin distribution delivered a very particular, and widely used, copula function. On top of other properties, which we do not discuss here, the main peculiarity is that it provides a dependence function that has both an absolutely continuous and a singular part. In plain terms, if we model survival times or any other set of variables, the Marshall–Olkin copula provides a positive probability that the event occurred at exactly the same time, or that the two variables that are being modelled take the same value. So, in a scatter diagram of a copula function there is a locus on which the points become more concentrated, designing a line with probability mass. This is obviously due to the idea of modelling an event that kills all the elements in the cluster, but it also conveys a property that is quite rare in the realm of copula functions. From the seminal work by Marshall and Olkin, a massive stream of literature has developed, trying to address the flaws of the model, mainly due to its very simple structure. Many of the papers included in this book are devoted to extensions of the model in several directions.

The main extensions are reviewed in the paper by Bernard, Fernàndez, Mai, Schenk and Scherer (BFMSS) in this book. This review, starting with the standard Marshall–Olkin model, describes the main alternative strategies to generate the same distribution by also providing novel representations. In general, the model can be extended in three main directions. One of the first issues that were raised about the model is that all shocks, both specific and systemic, are assumed to be independent. We could call this a pure systemic risk model. In the real world, and particularly in human sciences applications, sometimes disasters, that is failure of the whole system, are triggered by individual failures. A famous solution, reported in the BFMSS review, is the Lévy-frailty model. It is well known that frailty models lead to Archimedean dependence structures. Beyond this benchmark solution, other contributions in this book provide alternative derivations and extensions. Mulinacci proposes a power mixture approach that achieves a similar result of inducing dependence among the shocks in the model. Frostig and Pellerey suggest a dynamics for the common frailty variable. Augusto and Kolev propose a model in which the specific shocks are linked by a dependence structure, while the systemic shock is assumed to be independent.

The model can be extended in a second important direction, with the aim of embedding it in the standard common factor models that we find in linear statistics. The question is that in principle the Marshall–Olkin model, in which the observed variable is the minimum of a idiosyncratic and a systemic variable, is not very different from the standard factor model in which the observed variable is the sum, or an affine function of the two components. Of course, the difference stems from the different variables to which the models are applied. In the Marshall–Olkin model the variables are usually interpreted as lifetimes, are naturally defined on the positive line support, and are assigned an exponential distribution. Instead, in the standard linear factor model application to the stock market the variables are assumed to be returns, defined on the whole line support, and endowed with an elliptical distribution. Therefore, an interesting research question is what results are common to the use of different aggregation functions. This generalization is employed both in the contributed paper by Frostig and Pellerey and that by Durante, Girard and Mazo (DGM). Moreover, the paper by Augusto and Kolev explicitly addresses a natural dual model with respect to the Marshall–Olkin one that arises when the observed variable is the maximum of the idiosyncratic and the common component.

A third line of extension is the dimension of the clusters, namely the number of variables involved. This is the most severe limitation and the most challenging development in the model. In fact, the Marshall–Olkin model suffers from what is called the curse of dimensionality. Increasing the scale of the model very soon induces a degree of combinatorial complexity that is hard to tackle. The solution to the problem is twofold: either we accept to overlook the differences among the individuals and settle for an exchangeable approach or we reduce the number of clusters, by accepting to set to zero some common intensity. In the first case, we preserve the entire spectrum of clusters, but within each one of them, we only consider the average individual. In the second case, we may induce some misspecification in the dependence structure, since the degree of dependence induced by the clusters that are dropped from the analysis may induce a bias in the estimation of dependence in the remaining clusters. Extensions in the direction of non-exchangeability are recalled in the BFMSS paper, and a specific strategy to build models in the Marshall–Olkin spirit, with a focus on estimation issues, is addressed by DGM.

While the contributions in this book are mathematical and statistical in nature, the questions and the extensions addressed have relevant implications for economics, finance and politics. In fact, the economic and financial crisis is the main reason why the concepts of systemic risk and contagion have become paramount in this century. The two concepts have not been actually very well distinguished in public debate. For example, even in the definition of SIFIs, that is systemically important financial institutions, we actually refer to financial entities whose default may bring about a general crisis of the whole system: but this is actually contagion. The effects that followed the default of Lehman Brothers, on 15 September 2008, and that have persisted for years, represent a case of a systemic crisis that was actually triggered by the default of a component of the system.

From the point of view of economic policy and regulation, it is very important to distinguish systemic risk and contagion. Actually, the concept of pure systemic risk coincides with the original Marshall–Olkin framework, in which the common shock affecting all the components of the system is independent of the specific components. It is an act of God of which the components do not take the blame. For this reason, it is quite natural that the effects of this kind of events should be borne out by the community. In other words, a pure systemic event in the Marshall–Olkin spirit is like a natural catastrophe that the community is called to face.

Contagion is different: it is when the collapse of a component of the system brings about the default of a set of other components. Then, the Marshall–Olkin model extensions that drop the independence assumption of the shocks introduce an element of contagion. Each component may play a role in the default of the system as a whole, namely every component may trigger a systemic crisis. From a regulatory point of view, it is quite clear that the natural conclusion is different from that of a pure systemic risk event. More precisely, the expected cost of a systemic crisis triggered by one of the components should be borne out by the components themselves. They must be taxed, instead of the community, to make sure that they provide insurance for the damage that they may cause to the community. It is the principle known as "polluter must pay". Of course, non-exchangeability is also a paramount feature to allow for the application of the model. Consider again the problem that is more fashionable in these days, namely what is the impact of default of a single bank in the rest of the financial system. In addition, what about a major non-financial corporate entity? How do these events change the dependence structure of the remaining risks in the system? For some of these questions we have empirical evidence or at least case studies.

We have seen how the Lehman case was a disaster, bringing about contagion both in the US market and overseas, and not only to the rest of the financial system, but also to the sovereign entities. We also witnessed cases in which a credit event of a large corporate, such as the downgrading of General Motors below the investment grade rating line, was associated to a strong decrease in default dependence. In a pure systemic risk world, all these events should leave the rest of the system, and its dependence unaffected. In a world of exchangeable risk, the default of each element of the system would have the same impact both on the default probability of the other elements in the system and on the dependence structure of the remaining elements. Therefore, removing exchangeability from the system can be considered the next step beyond removal of the independence assumption.

Finally, the generalization of aggregation operators applying to idiosyncratic and systemic unobserved components is also an interesting topic to be discussed in practical applications. On the one hand, it would be interesting to discuss the economic meaning of different aggregation functions: when it has to be linear and which other specific shapes it has to take in other different hidden factor decompositions. On the other hand, there is an interesting question about how the aggregation operators linking hidden factors can be composed with aggregation operators applied to the observed variables. So, for example in standard statistics we have linear systems of observed variables that are in turn a linear combination of unobservable linear factors. In risk management, instead, it also well known that different aggregation operators applied to the risk measure, such as for example the Value-at-Risk, responds to different purposes. Therefore, the sum operator leads to the aggregation of the risk measure: one has to compute the risk measure of the convolution of the risk exposures. Differently, using the max aggregator provides an answer to risk capital allocation, and can be applied to studying the trade-off among different exposures.

In the future, it will be very interesting to address these problems for the Marshall–Olkin model and its extensions. The interest mainly stems from the peculiarity of the dependence structure, and the question of how the singularity in it could affect the results. A first interesting contribution in this direction is provided by Fernandez, Mai and Scherer in their contribution in this book. It addresses the computation of the convolution of random variables linked by Marshall–Olkin dependence, in the simple model of exchangeable risks, finding that in this case one can recover analytical formulas for systems of dimension up to four.

In conclusion, we hope that the results reported in this book could represent a step of development in this interesting field of research started by Marshall and Olkin in the early second half of the past century, and so modern with respect to the main issues raised by the globalisation of risks.

In the end, we express our gratitude to all the people involved in the realization of this project. Moving backward, we would like to thank the referees, both internal and external, for their excellent job in reviewing the papers collected in this book. We thank the authors of the contributed papers for their patience and their timeliness to provide the paper. We thank the Graduate Course in Quantitative Finance for providing the funds for the organization of the international conference where

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