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Linear and Mixed Integer Programming for Portfolio Optimization

 Springer

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To Giovanni and Matteo
To Barbara and Bartek
To Massimo, Chiara, and Laura

Preface

Portfolio theory was first developed by Harry Markowitz in the 1950s. His work, which was extended by several researchers, provides the foundation of the so-called *modern portfolio theory*. Markowitz work has been published and discussed in several papers and books. He introduced the concept of diversification and captured in a model the importance of investing in a diversified portfolio. His pioneering model has the goal to find the optimal trade-off between the risk and the return of an investment. The risk of the portfolio is measured through the variance of the portfolio rate of return. The resulting optimization model, which is a single period model for portfolio selection, is quadratic. In the last two decades, several models have been proposed for portfolio optimization that use different (with respect to the variance) functions to measure the performance of a portfolio. Several resulting optimization models are linear. Linear programming models open up the possibility to consider additional features of the investment problem, also those that imply the introduction of binary and integer variables.

The optimization models for portfolio selection that are the focus of this book are of interest in many application areas, from finance to business and engineering. Although we use the basic language of finance and talk about the investment of a capital in assets, the models can be applied to a broad variety of portfolio selection problems.

The book presents the general problem of single period portfolio optimization, the different linear models arising from different performance measures, and the mixed-integer linear models resulting from the introduction of real features. Transaction costs may be charged when a specific asset or a group of assets is included in the portfolio. The presence of such costs is among the most important features of the real problems that, to be embedded in an optimization model, require in most cases binary variables. Other linear models, such as models for portfolio rebalancing and index tracking, are also covered. Computational issues are discussed and the theoretical framework, including the concepts of risk averse preferences, stochastic dominance, and coherent risk measures, is provided.

The material in the book is presented assuming that the reader does not have a background in finance or in portfolio optimization. Only some experience in linear

and mixed-integer models is assumed. The material is presented in a didactic way. Concepts are accompanied by comments and examples. This is in a sense the book we wish we had when we first started working on portfolio optimization, coming from education in operations research. The target readers of this book are students – undergraduate, MBA, PhD – researchers, and professionals interested in the use of optimization models for portfolio selection.

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