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Spherical Radial Basis Functions, Theory and Applications

Springer

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Simon *dedicates this book to his partner Michelle and his two daughters Nancy and Clara.*

Thong *dedicates this book to his wife Tram Linh*.

Tanya *dedicates this book to her husband Chris and her two sons Kai and Eldin.*

Preface

In recent years mathematicians and researchers within the approximation theory community have become increasingly interested in using tools from approximation theory to develop numerical methods for problems set on spheres. Given that the two-dimensional sphere serves as a model for the surface of the earth, these problems are often generated by real-world applications. For instance, as more and more satellites are launched into space, the acquisition of global data is becoming more widespread and hence there is now increased demand for spherical data processing solutions. Another important application is in the quest to improve weather forecasting models through the development of efficient algorithms for solving partial differential equations (PDEs) posed on the surface of the sphere. There is also need for spherical approximation in areas other than geoscience and meteorology. For instance, one may want to construct a smooth surface that encloses a cloud of scattered points in Euclidean space, this can be achieved using spherical approximation solutions. This has numerous applications, for example, to model 3D objects for computer graphics, or to create a boundary model representing the safe operating envelope of internal combustion engines.

This book serves to provide an introduction to the theory and applications of spherical (radial) basis functions (SBFs), which represent one of the most promising emerging technologies for solving spherical problems. SBFs are closely related to the more famous family of radial basis functions (RBFs) which are already well-established tools for solving data fitting problems and PDEs over regions in Euclidean space. RBFs have a much longer history than SBFs and so, consequently, much more is known about them, indeed [Buh03; Fass07; Wen05] are three excellent textbooks devoted to their theoretical properties and their practical implementations. Our primary aim in this book is to present enough practical and theoretical details to enable the reader to implement SBF techniques to solve real problems and also, if desired, to pursue further theoretical studies in this exciting area. In Chap. 1 we set out our motivation for studying SBFs and provide the background tools from functional analysis which will be used throughout the book. In Chap. 2 we demonstrate how key ideas and concepts from the interpolation theory of RBFs in Euclidean space can be recast into the spherical setting and, in doing so, we introduce the notion of SBFs and we show how they can be used to provide unique solutions (SBF interpolants) to data fitting problems on the sphere. Furthermore, we also reveal a simple variational framework for SBF interpolation and show how this can be used to analyse the accuracy of a particular SBF interpolant to a given target function. In Chap. 3 we pursue the error analysis in much greater detail. Specifically, we present the technical ingredients of an error bounding strategy which we then use to provide much improved error estimates for SBF interpolation. In Chap. 4 we test the theory and present the results of numerical experiments for the SBF method for solving data fitting problems on the sphere. In the final two chapters of the book we move away from data fitting applications and concentrate more on investigating how SBF approximations can be used to solve PDEs on spheres. In Chap. 5 we focus more on computational issues and propose a preconditioning strategy to speed up the iterative solution of an elliptic PDE. Finally, in Chap. 6, we examine the inhomogeneous heat equation as an example of a parabolic PDE. Here we develop a collocation solution method and provide a full error analysis when the time variable is discretized using either the backward Euler or the Crank-Nicolson method.

In summary, the material covered in this book is aimed at graduate students and researchers in mathematics and related fields such as the geophysical sciences and statistics. We have tried to make the exposition as clear and as self-contained as possible and have made efforts to ensure that technical details are explained in a friendly and readable style. We hope this will encourage the reader to delve deeper and discover more.

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