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Editors

Correlated Random Systems: Five Different Methods

CIRM Jean-Morlet Chair, Spring 2013

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Preface

The Jean Morlet Chair is a recently installed initiative by the CIRM in Luminy (CNRS-SMF), Aix Marseille University, and the city of Marseille. Each year, two chairs are assigned. The chairholder, together with a local organizer, is then given the opportunity to run a research program, which includes a series of conferences, lectures, and seminars. The first Jean Morlet Chair took place in the first half of 2013, and was devoted to probability. Nicola Kistler (Frankfurt) was chairholder, and Véronique Gayrard (AMU-CNRS) acted as local organizer. In this volume, one finds the notes of five lectures which were delivered within this program by Erwin Bolthausen, Francesco Guerra, Nicola Kistler, Jay Rosen, and Benjamin Schlein.

It is challenging to find a *thematic* thread between the five lectures. Rather, we would like to see this collection as an account of five *tools*, which have recently emerged in the study of highly correlated random systems. These tools are, respectively: (a) the large deviation principle for words for the study of random polymers, (b) the interpolation techniques for the study of mean field spin glasses, (c) the second-moment method for the extremes of combinatorial structures driven by multiple scales, (d) the Dynkin isomorphisms for the study of local times and their relation with loop soups, and (e) the local convergence of the density of states and the local relaxation flow in the context of random matrices.

Current research in probability deals with models, often stemming from mathematical physics, where classical results such as the law of large numbers or the central limit theorem no longer hold as a consequence of severe correlations. In case the assumption of independence is no longer met, it is clearly unreasonable to expect "universally-valid" methods, and one has to proceed with a model-dependent analysis. However, the recent astonishing activity in and around the fields covered by the contributions to this volume suggests that it is still possible to develop powerful tools whose range of applications may go well beyond the original setting. For instance, the large deviation principle for words has found important applications in the analysis of collision local times of transient random walks, or in the study of intermediate phases in interacting stochastic systems. The interpolation method, at first introduced in the study of the Sherrington-Kirkpatrick model, has become meanwhile a fundamental tool in theoretical computer science. The multiscale refinement of the second moment method (and versions thereof) first emerged in the study of the extremes of branching Brownian motion has been recently applied to the study of the two-dimensional Gaussian free field, and log-correlated random fields. Dynkin isomorphisms have re-gained momentum due to their prominent role in the recent development of cover times. Finally, the revolution sparked by the convergence of the density of states and the local relaxation flow in random matrices hardly needs any comment.

Literally hundreds of papers resort, in one way or another, to the five tools presented in these lectures notes. Given this state of affairs, it will not come as a surprise that the lectures are by no means self-contained. The material is presented in a rather informal style which strives to convey the main ideas without entering into the burdening technical issues lurking behind the surface. As such, the contributions to this volume should be seen as a first exposure to five tools which have become indispensable in the study of correlated random systems.

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Frankfurt am Main, Germany Nicola Kistler

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