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Volume 1 provides a series of lectures by internationally well-known authors based on the year on Stochastics in biological systems which took place at the MBI in 2011–2012.

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**Mathematical Biosciences Institute Lecture Series**
**Volume 1: Stochastics in Biological Systems**

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# Branching Process Models of Cancer

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# Preface

In this chapter, we will use multitype branching processes with mutation to model cancer. With cancer progression, resistance to therapy, and metastastis in mind, we will investigate $\tau_k$, the time of the first type $k$ mutation, and $\sigma_k$, the time of the first type $k$ mutation that founds a family line that does not die out, as well as the growth of the number of type $k$ cells. The last three sections apply these results to metastasis, ovarian cancer, and tumor heterogeneity. Even though martingales and stable laws are mentioned, these notes should be accessible to a student who is familiar with Poisson processes and continuous time Markov chains.

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Durham, NC, USA Richard Durrett

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