Solid Mechanics and Its Applications

Volume 219

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Fractional Thermoelasticity

Springer

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Preface

Thus, when God said
Let there be light, He implied,
Let there also be heat—
and there was heat.
I. McNeil
What would physics look like without gravitation?
Albert Einstein
What would physics look like without heat conduction?
Yuriy Povstenko

The famous Fourier law, which states the linear dependence between the heat flux vector and the temperature gradient, was formulated by Fourier in 1822 and marked the beginning of the classical theory of heat conduction. A few years later, Fourier's disciple Duhamel coupled the temperature field and the body deformation and pioneered studies on thermoelasticity.

The classical theory of heat conduction based on the phenomenological Fourier law, which ignores processes occurring at the microscopic level, is quite acceptable for different physical situations. However, many theoretical and experimental studies of transport phenomena testify that in media with complex internal structure (amorphous, porous, random and disordered materials, fractals, polymers, glasses, dielectrics and semiconductors, etc.) the classical Fourier law and the standard parabolic heat conduction equation are no longer accurate enough, and physical processes occurring at the microscopic level, in one way or another, should be taken into account. This leads to formulation of nonclassical theories, in which the Fourier law and the parabolic heat conduction equation are replaced by more general equations.

Each generalization of the heat conduction equation results in formulation of the corresponding generalized theory of thermal stresses. For example, thermoelasticity without energy dissipation proposed by Green and Naghdi [1] is based on the wave equation for temperature. Cattaneo's telegraph equation for temperature leads to the generalized thermoelasticity of Lord and Shulman [2]. This book is devoted to fractional thermoelasticity, i.e., thermoelasticity based on the heat conduction equation with differential operators of fractional order. Time-fractional differential operators describe memory effects, space-fractional differential operators deal with the long-range interaction. It should be emphasized that fractional calculus has been successfully used in physics, geology, chemistry, rheology, engineering, bioengineering, robotics, etc. The first paper on fractional thermoelasticity was published by the author in 2005. During the last decade, substantial literature on this subject has evolved, but there is no book which sums up investigations in this field. The present book, which for the major part is based on the author's research, fills in such a blank.

The book is organized as follows. Chapter 1 presents essentials of fractional calculus. Different kinds of integral and differential operators of fractional order are discussed (the Riemann-Liouville fractional integrals, the Riemann-Liouville and Caputo fractional derivatives, and the Riesz fractional operators). Chapter 2 is devoted to time- and space-nonlocal generalizations of the Fourier law, the corresponding generalizations of the heat conduction equation, and formulation of associated theories of fractional thermoelasticity. Different kinds of boundary conditions for the time-fractional heat conduction equation are analyzed including the conditions of perfect thermal contact and the moving interface boundary conditions at the solid–liquid interface. In Chaps. 3 and 4 the axisymmetric problems for the time-fractional heat conduction and associated thermal stresses are considered in polar and cylindrical coordinates, respectively. The central symmetric problem in spherical coordinates are studied in Chap. 5. It should be noted that the considered theory interpolates the classical theory of thermal stresses based on the parabolic heat conduction equation and the theory of thermoelasticity without energy dissipation proposed by Green and Naghdi and started from the hyperbolic wave equation for temperature. Chapter 6 presents thermoelasticity based on the space-time-fractional heat conduction equation. Chapter 7 is devoted to thermoelasticty which uses the fractional telegraph equation for temperature (fractional generalization of the well-known theory of Lord and Shulman). In Chap. 8 we formulate equations of fractional thermoelasticity of thin shells (solids with one size being small with respect to two other sizes). The generalized boundary conditions of nonperfect thermal contact for the time-fractional heat conduction in composite medium are also formulated. It is well known that from the mathematical viewpoint, the Fourier law and the theory of heat conduction and the Fick law and the theory of diffusion are identical. Chapter 9 deals with the theory of diffusive stresses caused by fractional advection-diffusion equation.

The book contains a large number of figures which show the characteristic features of temperature and stress distributions and represent the whole spectrum of order of fractional operators.

The corresponding sections of the book may be used by university lecturers for courses in heat and mass transfer, continuum mechanics, thermal stresses, as well as in fractional calculus and its applications for graduate and postgraduate students. The book presents a picture of the state of the art of fractional thermoelasticity and will also serve as a reference handbook for specialists in applied mathematics, physics, geophysics, elasticity, thermoelasticity, and engineering sciences. The book provides information which puts the reader at the forefront of current research in the field of fractional thermoelasticity and is complemented with extensive references in order to stimulate further studies in this field as well as in the related areas.

Częstochowa, November 2014 Yuriy Povstenko

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