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Yasumichi Hasegawa

# Control Problems of Discrete-Time Dynamical Systems

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# Preface

This monograph deals with control laws of discrete-time dynamical systems which include linear and nonlinear input/output relations. It will be of popular interest to researchers, engineers and graduate students who specialized in system theory. In the monograph [Hasegawa, 2013], new control problems were proposed with a point of the following rough view:

After a state's being controlled to any state, we may have a result which says that the state can be only controlled to the same state once every  $n$  sampling times in a  $n$ -dimensional canonical system. That means that any state does not remain static on different time zone. From the point of view that control is, roughly speaking, putting something in motion to our satisfaction, the change of the state in discrete-time system may not be considered good for the concept of control. Therefore, the state control problem is not suitable for discrete-time dynamical systems except equilibrium state control. Thus in monograph [Hasegawa, 2013], we showed that a state control problem is nonsense except equilibrium state control problem in discrete-time dynamical systems and we showed that the control problem is suitable for the output control which is the fixed value output control or the tracking output control and we showed the existence of solutions by using a nonlinear programming for our solving control problems.

This monograph is the second enlarged edition of the monograph [Hasegawa, 2013].

From the very new point, the problem will be treated in the sense of time-domain approach. A new method which produce manipulated inputs will be proposed in the sense of equilibrium state control and output control and be represented as a state at the preceding time and a desired output value at the same point in time.

Here we want to remember that the many results obtained in continuous-time optimal control were only converted to appropriate forms in discrete-time optimal control without using the special properties in discrete-time case.

Using the method thus allows control inputs to be induced by characteristic phenomena of discrete-time and canonical finite dimensional dynamical systems. By virtue of this approach, this monograph provides new control laws and their extensions which can also be more applicable for nonlinear dynamical systems.

To present the effectiveness of our method, many numerical examples of control problem will also be provided.

In analysis of state space approach, control problem may have become a theme of technology after 1960 for the purpose of efficiency in the field of economy, industrial technology and others.

Usual modern control design requires the solution of complicated nonlinear matrix equations, on the other hand it is lacking in some aspects. The designed performance obtained by solving matrix design equations means that it is often possible to design a control system that works in theory without gaining any engineering intuition about the problem.

But our proposal provides a sort of intuition which means the closeness to input, output and the state. And we will solve our control problems as algebraically as possible for the first time.

Note that we could discuss our control problems in the sense of non-linear programming in the monograph [Hasegawa, 2013].

It is well known that a state of dynamical systems may be easily changed into another state by the free motion or inputs except equilibrium state and also known that the output value of a state is very different from the output value of the preceding state. If we want to construct a control system to be smooth and gradual, we must consider output control problem. Therefore, in the sense of input and output, control problem of output is more practical than control problems of input and state which are used in the usual control problem.

Our proposal needs a computer-aided design which is an essential feature of modern controls.

Be based on input output control, our control problems for a given dynamical system with input and output can be roughly stated as the following three problems:

1: *equilibrium state control*

Find an input sequence that will bring an arbitrary state of the system to the equilibrium state (especially in linear system, the zero state) within the size of input values.

2: *fixed value output control*

Find an input sequence that will bring an arbitrary output of the system to fixed value output within the size of input values.

3: *tracking output control*

Find an input sequence that will bring an arbitrary output of the system to a desired trajectory output within the size of input values.

It is worth to remember that the development of control problem has been strongly stimulated by linear system theory well-connected with the development of digital computers and related mathematics, for example, matrix theory or mathematical programming. However, such development of nonlinear dynamical systems has not been occurred yet because there have been no suitable mathematical method for nonlinear systems, for example without utilizing characteristic phenomena of discrete-time and finite dimensional dynamical systems.

In this monograph, regarding the output sequence to be controlled as the equations to be expressed by terms of input, we identify our control problem as a problem of finding the unique inputs which produce the specified output. If we have not obtained the unique input, we will obtain the unique input with introducing the performance function for inputs to be treated as the square norm, namely, in the sense of energy. Our method intensively takes a positive attitude toward using computers. Consequently, we will introduce a method called *least square method in the sense of energy*.

In this monograph, we will also discuss adaptive control problems in the sense of input output control based on an algebraic control solution for the first time.

The problem can be stated as follows:

For a given unknown system with only known dimensional number of the state space in the system, find the system and solve our input output control problems.

It is very important that the adaptive control problems can be solved as easily as possible. We can solve the problem for systems which include linear systems, so-called linear systems, almost linear systems and pseudo linear systems because the systems satisfy the time-invariance.

Because general non-linear systems, e.g, affine dynamical systems and linear representation systems do not satisfy the time-invariance, we cannot solve the new adaptive control problem.

As already mentioned, the usual control problem has been mainly discussed in linear systems. On the other hand, there are few developments for nonlinear systems. Our recent monograph *Realization Theory of Discrete-Time Dynamical Systems* (T. Matsuo and Y. Hasegawa, Lecture Notes in Control and Information Science, Vol. 296, Springer, 2003) indicated that any input/output map of nonlinear dynamical systems can be characterized by Hankel matrix or Input/output matrix, which are very similar to Hankel matrix in linear systems. The monograph also presented that obtaining a dynamical system which describes a given input/output map is equal to determining the rank of the matrix of the input/output map and the coefficients of a linear combination of column vectors in the matrix. We know that the reachability means the controllability in discrete-time systems and that the reachability is completely determined by the rank of a matrix. And for observability, we may be similar in concept. These insight leads to the ability of discussing fruitful control problem, especially for both linear and non-linear dynamical systems.

Note that general non-linear systems, e.g, affine dynamical systems and linear representation systems can not be expressed as Hankel matrix or Input/output matrix.

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We also wish to thank Professor R. E. Kalman for his suggestions.

He stimulated us to research these problems of system theory directly as well as through his works.



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