

UNITEXT – La Matematica per il 3+2

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Mathematical Analysis I

Second Edition

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Preface

This textbook is meant to help students acquire the basics of Calculus in curricula where mathematical tools play a crucial part (so Engineering, Physics, Computer Science and the like). The fundamental concepts and methods of Differential and Integral Calculus for functions of one real variable are presented with the primary purpose of letting students assimilate their effective employment, but with critical awareness. The general philosophy inspiring our approach has been to simplify the system of notions available prior to the university reform; at the same time we wished to maintain the rigorous exposition and avoid the trap of compiling a mere formulary of ready-to-use prescriptions.

In view of the current Programme Specifications, the organization of a first course in Mathematics often requires to make appropriate choices about lecture content, the comprehension level required from the recipients, and which kind of language to use. From this point of view, the treatise is ‘stratified’ in three layers, each corresponding to increasingly deeper engagement by the user. The intermediate level corresponds to the contents of the eleven chapters of the text. Notions are first presented in a naïve manner, and only later defined precisely. Their features are discussed, and computational techniques related to them are exhaustively explained. Besides this, the fundamental theorems and properties are followed by proofs, which are easily recognisable by the font’s colour.

At the elementary level the proofs and the various remarks should be skipped. For the reader’s sake, essential formulas, and also those judged important, have been highlighted in blue, and gray, respectively. Some tables, placed both throughout and at the end of the book, collect the most useful formulas. It was not our desire to create a hierarchy-of-sorts for theorems, instead to leave the instructor free to make up his or her own mind in this respect.

The deepest-reaching level relates to the contents of the five appendices and enables the strongly motivated reader to explore further into the subject. We believe that the general objectives of the Programme Specifications are in line with the fact that willing and able pupils will build a solid knowledge, in the tradition of the best academic education. The eleven chapters contain several links to the different appendices where the reader will find complements to, and insight in

various topics. In this fashion every result that is stated possesses a corresponding proof.

To make the approach to the subject less harsh, and all the more gratifying, we have chosen an informal presentation in the first two chapters, where relevant definitions and properties are typically part of the text. From the third chapter onwards they are highlighted by the layout more discernibly. Some definitions and theorems are intentionally not stated in the most general form, so to privilege a brisk understanding. For this reason a wealth of examples are routinely added along the way right after statements, and the same is true for computational techniques. Several remarks enhance the presentation by underlining, in particular, special cases and exceptions. Each chapter ends with a large number of exercises that allow one to test on the spot how solid one's knowledge is. Exercises are grouped according to the chapter's major themes and presented in increasing order of difficulty. All problems are solved, and at least half of them chaperone the reader to the solution.

We have adopted the following graphical conventions for the constituent building blocks: definitions appear on a gray background, theorems' statements on blue, a vertical coloured line marks examples, and boxed exercises, like 12., indicate that the complete solution is provided.

We wish to dedicate this volume to Professor Guido Weiss of Washington University in St. Louis, a master in the art of teaching. Generations of students worldwide have benefited from Guido's own work as a mathematician; we hope that his own clarity is at least partly reflected in this textbook.

This second English edition reflects the latest version of the Italian book, that is in use since over a decade, and has been extensively and successfully tested at the Politecnico in Turin and in other Italian Universities. We are grateful to the many colleagues and students whose advice, suggestions and observations have allowed us to reach this result. Special thanks are due to Dr. Simon Chiossi, for the careful and effective work of translation.

Finally, we wish to thank Francesca Bonadei – Executive Editor, Mathematics and Statistics, Springer Italia – for her encouragement and support in the preparation of this textbook.

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