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Introduction to the Analytic Hierarchy Process

 Springer

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Preface

Why would anyone feel urged to write another book on the Analytic Hierarchy Process (AHP), given those already written? I felt urged because the existing books on the AHP are conservative, too anchored to the original framework and do not cover recent results, whereas lots of questions have been addressed in the last years. Apparently, the interest in the AHP has not faded in the last years, and we shall see that this view is also supported by other studies, as well as by the years of publication of many of the references used in this booklet.

Now, the next question one should ask himself when writing a tutorial should regard to whom the tutorial is for. With the premise that a decision scientist might find these pages too simplistic, in my intentions, the readership should include the following categories.

- *Practitioners* and *consultants* willing to apply, and *software developers* willing to implement, the AHP. Some collateral issues, for instance the incompleteness of judgments, are usually neglected in didactic expositions, but remain fundamental in practical implementations. On the software development side, at present, there is still not a modern and free software which covers all the aspects of the AHP presented in this booklet
- Recent advances in the theory have been disseminated in different journals and, as research requires, are narrow, technical, and often use heterogeneous notation and jargon. Therefore, I also hope that *students* who have been introduced to the AHP and want to have an updated exposition on, and references to, the state of the art can find these pages useful
- Even the *applied mathematicians* might find it interesting. The mathematics behind the method is simple, but some of its extensions have been a fertile ground for the application of non-trivial concepts stemming from abstract algebra and functional analysis, just to mention two areas of interest.

The following pages assume neither previous knowledge of the AHP, nor higher mathematical preparation than some working knowledge of calculus and linear algebra with eigenvector theory. A brief tutorial on eigenvalues and eigenvectors is provided in the appendix. Moreover, some sections are marked with the symbol * to

indicate that they contain further discussions and references to research literature. The reader interested in the fundamentals might want to skip them.

Ideally, this booklet is also articulated to suit different levels of readership. I believe that the following three can serve as approximate guidelines:

- A *basic* exposition is given in Chapter 1 with the exclusion of the section marked with *. The reader can then proceed examining Section 2.1 until the end of §2.1.1, Section 2.2 until the end of §2.2.1, and Section 3.2 with the exclusion of the subsection marked with *. A basic understanding allows the reader to use the AHP only at a superficial level.
- A *complete* exposition of the AHP can be gained by reading this booklet in its entirety, with the exclusion of the sections marked with *. A complete understanding allows the reader to choose between different tools to perform different tasks.
- An *advanced* understanding of the method is like the complete, but with the addition of the sections marked with *. Compared to the complete understanding, in the advanced, the reader will familiarize with the most recent results and the ongoing discussions, and will be able to orient through the literature.

I shall also spell out that I will not refrain from giving personal perspectives onto some problems connected with the AHP, as the method has been a matter of heated debate since its inception.

I hereby wish to thank those who helped me. Among them, I am particularly grateful to Michele Fedrizzi, who also taught me much of the material contained in this booklet. I am also grateful to Springer, especially in the person of Matthew Amboy. Furthermore, this project has been financed by the Academy of Finland.

It goes without saying that I assume the paternity of all imprecisions and mistakes and that the reader is welcome to contact me.

Espoo, Finland, November 2014

Matteo Brunelli

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