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Volume 27

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Vigen Arakelian • Sébastien Briot

# Balancing of Linkages and Robot Manipulators

Advanced Methods with Illustrative  
Examples

 Springer

Vigen Arakelian  
Institut National des Sciences  
Appliquées (INSA), Rennes, France,  
and Institut de Recherche en  
Communications et Cybernétique  
de Nantes (IRCCyN), Nantes, France  
Rennes  
France

Sébastien Briot  
Institut de Recherche en Com.  
et Cybernétique de Nantes  
CNRS  
Nantes  
France

ISSN 2211-0984  
Mechanisms and Machine Science  
ISBN 978-3-319-12489-6  
DOI 10.1007/978-3-319-12490-2

ISSN 2211-0992 (electronic)  
ISBN 978-3-319-12490-2 (eBook)

Library of Congress Control Number: 2014958013

Springer Cham Heidelberg New York Dordrecht London  
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*Vigen Arakelian dedicates this work to his charming wife Tatiana and beloved son David.*

*Sébastien Briot dedicates this work to his beloved wife and sons, Sylvie, Élouan and Guénaël.*

# Preface

The balancing of linkages is an integral part of the mechanism design. The challenge of reducing vibrations of the frame on which the mechanism is mounted is nothing new. Despite its long history, mechanism balancing theory continues to be developed and new approaches and solutions are constantly being reported. Hence, the balancing problems are of continued interest to researchers. Several laboratories around the world are very active in this area and new results are published regularly. In recent decades, new challenges have presented themselves, particularly, the balancing of robots for fast manipulation.

The authors believe that this is an appropriate moment to present the state of the art of the studies devoted to balancing and to summarize their research results. This monograph is based on the material published by the first author over the last twenty years and the doctoral dissertation of the second author defended in 2007 and rewarded by the Research Group in Robotics of the French National Center for Scientific Research (GDR Robotique, CNRS, 2008), the French Section of the ASME (2011) and the French Région Bretagne in the category “Sciences, Technologies and Interdisciplinarity” (2011).

Some results given in the book were reached in collaboration with Mike Smith, Clément Gosselin, Ilian Bonev, Simon Lessard and Cédric Baradat. The authors acknowledge for their contributions, as well as the “Mechanical Center” of the National Institute of Applied Sciences of Rennes for the development of the prototypes permitting the validation and improvement of the obtained theoretical results.

The authors will be also genuinely grateful to the readers for any critical remarks on the material presented in the book and for any suggestion for its improvement.

Rennes/Nantes, France  
June, 2014

Vigen Arakelian  
Sébastien Briot

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# List of Symbols and Abbreviations

In the whole book, vectors are represented by bold lowercase symbols and matrices by bold uppercase symbols, except for greek symbols.

## List of Symbols

$\alpha, \beta, \gamma, \phi, \varphi, \theta, \psi$	angles used for geometry description of a mechanism
<b>f</b>	a vector of force
<b>f<sup>sh</sup></b>	a vector of shaking force
<i>F</i>	a force
<i>F<sup>sh</sup></i>	a shaking force
<b>h</b>	an angular momentum
$H_x, H_y, H_z$	the components of the angular momentum <b>h</b> around <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
<b>I<sub>j</sub></b>	the inertia matrix for body <i>j</i> , expressed at the <i>com</i> in the local frame attached to this body
$I_{cr_j}$	the axial moment of inertia of the counter-rotation <i>j</i>
$I_{S_j}$	the axial moment of inertia of the link <i>j</i> expressed at the <i>com</i> when link <i>j</i> is considered to have a planar motion
$I_{xx}^{(j)}, I_{yy}^{(j)}, I_{zz}^{(j)}$	the axial moments of inertia around <b>x</b> , <b>y</b> and <b>z</b> axes, respectively, for body <i>j</i> , expressed at the <i>com</i> in the local frame attached to this body
$I_{xy}^{(j)}, I_{yz}^{(j)}, I_{yz}^{(j)}$	the inertial cross-moments around <b>z</b> , <b>y</b> and <b>x</b> axes, respectively, for body <i>j</i> , expressed at the <i>com</i> in the local frame attached to this body
<i>L</i>	the Lagrangian of a system
$l_{PQ}$	the length of the segment <i>PQ</i>
<b>m</b>	a vector of moment
<i>M</i>	a moment
<b>m<sup>sh</sup></b>	a vector of shaking moment
<i>M<sup>sh</sup></i>	a shaking moment
$m_j$	the mass of the body <i>j</i>
$m_{cw_j}$	the mass of the counterweight <i>j</i>
$\omega_j$	the rotational velocity of body <i>j</i> expressed in the base frame

$\omega_{j_x}, \omega_{j_y}, \omega_{j_z}$	the components of the vector $\omega_j$ around <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
$\dot{\omega}_j$	the rotational acceleration of body $j$ expressed in the base frame
$\dot{\omega}_{j_x}, \dot{\omega}_{j_y}, \dot{\omega}_{j_z}$	the components of the vector $\dot{\omega}_j$ around <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
<b>p</b>	a linear momentum
$P_x, P_y, P_z$	the components of the linear momentum <b>p</b> along <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
<b>q, <math>\dot{q}</math>, <math>\ddot{q}</math></b>	vectors of actuated coordinates, velocities and accelerations, respectively
$q, \dot{q}, \ddot{q}$	some actuated coordinates, velocities and accelerations, respectively
${}^i\mathbf{R}_j$	the rotation matrix from the frame $i$ to the frame $j$
$\mathbf{r}_P$	the position of point $P$ expressed in the base frame
$\dot{\mathbf{r}}_P$	the velocity of point $P$ expressed in the base frame
$\ddot{\mathbf{r}}_P$	the acceleration of point $P$ expressed in the base frame
$S_j$	the <i>com</i> of the body $j$
$t$	the time variable
$T$	the kinetic energy of a system
<b>x</b>	for a manipulator, the Cartesian position of its end-effector
$x_P, y_P, z_P$	the components of the vector $\mathbf{r}_P$ along <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
$\dot{x}_P, \dot{y}_P, \dot{z}_P$	the components of the vector $\dot{\mathbf{r}}_P$ along <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
$\ddot{x}_P, \ddot{y}_P, \ddot{z}_P$	the components of the vector $\ddot{\mathbf{r}}_P$ along <b>x</b> , <b>y</b> and <b>z</b> axes, respectively
$\tau$	an input torque
$V$	the potential energy of a system
<b>w</b>	a wrench

### List of Abbreviations

<i>com</i>	centre of mass
<i>dof</i>	degree of freedom
HOBM	hand-operated balanced manipulators
PAMINSA	parallel manipulator of the INSA
<i>PKM</i>	parallel kinematic machine
<i>PPM</i>	planar parallel mechanism
<i>rms</i>	root mean square
<i>SPM</i>	spatial parallel mechanism