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Statistical Physics of Non-Thermal Phase Transitions

From Foundations to Applications

 Springer

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*“To those summer sunny days,
When world was warm and still,
And unicorn’s four gleamy eyes
Were made of glass and steel,
The running man was hunt in maze
To make a Minotaur’s meal,
But slow, emerald-green waves
Danded: “Drive the quill!”
A hurt white-crow made mistakes
Against its kind and will,
And near train depot earthquakes
Became a part of cozy home deal.
A life was crazy like a waste
Collecting future regrets’ bill.”*

Preface

Statistical physics describes a wide variety of phenomena and systems when interaction forces may have different natures: mechanical, electromagnetic, strong nuclear, etc. The commonality that unites all these systems is that their belonging to statistical physics requires the presence of thermal fluctuations. In this sense these phenomena necessarily include the thermodynamic aspect.

Meanwhile, the second half of the last century may be named the time of the discovery of the so-called complex systems. These systems belong to chemistry, biology, ecology, geology, economics, social sciences, etc. and are generally united by the absence of concepts such as temperature or energy. Instead, their behavior is governed by stochastic laws of nonthermodynamic nature; and these systems can be called nonthermal. Nevertheless, in spite of this principal difference with statistical physics, it was discovered that the behavior of complex systems resembles the behavior of thermodynamic systems. In particular, many of these systems possess a phase transition identical to critical or spinodal phenomenon of statistical physics.

This very analogy has led in recent years to many attempts to generalize the formalism of statistical physics so that it would become applicable and for nonthermal systems also. If we achieved this goal, the powerful, well-developed machinery of statistical physics would help us to explain phenomena such as petroleum clusters, polymerization, DNA mechanism, informational processes, traffic jams, cellular automata, etc. Or, better, we might be able to predict and prevent catastrophes such as earthquakes, snow-avalanches and landslides, failure of engineering structures, economical crises, etc.

However, the formalism of statistical physics is developed for thermodynamic systems; and its direct application to nonthermal phenomena is not possible. Instead, we first have to build analogies between thermal and nonthermal phenomena.

But, what do these analogies include? What are they based on? And even more important question: Why does the behavior of complex systems resemble their thermodynamic analogues?

The answer to the last question is that the analogy exists only in the presence of phase transitions. It is the machinery of a phase transition that is universal, not the systems themselves. In spite of the fact that the behavior of complex systems is governed by nonthermal fluctuations whose nature is quite different from thermal

fluctuations in statistical physics, these fluctuations are, nevertheless, stochastic and scale invariant; and it is the stochastic scale invariance of the system that leads to the universality of phase transitions. Therefore, our attempt to apply the formalism of statistical physics to nonthermal phenomena would be successful only if we mapped the nonthermal fluctuations on their thermal analogues.

This book is devoted to the comparison of thermal and nonthermal systems. As an example of a thermodynamic system we generally discuss an Ising model while the considered nonthermal systems are represented by percolation and damage phenomena. Step-by-step, from the equation of state to the free energy potential, from correlations to the susceptibility, from the mean-field approach to the renormalization group, we compare these systems and find that not only are the rules of behavior similar but also, what is even more important, the methods of solution. We will see that, developing the concept of susceptibility or building the renormalization group, although each time we begin with a particular system considered, the foundation of an approach is always based on the formalism of statistical physics and is, therefore, system independent.

To the purpose of comparison we often sacrifice in this book the specific details of the behavior of particular systems discussed. We cannot claim our study to be complete in the description of rigorous formalism or experimental results of ferromagnetic, percolation, or damage phenomena. Instead, we focus our attention on the intuitive understanding of the basic laws leading to the analogies among these systems. For the same reason and also because we consider our text to be introductory, we cannot claim our list of references to represent all corner-stone studies related to the discussed phenomena. Instead, we are generally referring the reader to the brilliant reviews and references therein.¹

Also, we should mention that, although in many aspects this book may represent the biased view of its author, we hope that the reader will enjoy, as we do, the mystery of the birth of a new science that has been happening right before our eyes during the last few decades. Since this new science, in our humble opinion, is still at the infantile stage, there are many questions in the book which we cannot answer. However, from our point of view this adds an additional charm to the discussion because it encourages the reader to generate and apply her/his own ideas at the frontiers of science.

Another important aspect of the book is that the comparison with nonthermal systems presents the alternative point of view on thermodynamic phenomena themselves. Not all concepts of statistical physics have their counterparts in complex systems. Thereby, nonthermal phenomena often allow looking at well-known phenomena from quite a different angle to emphasize the omissions in statistical physics itself.

¹ The author would appreciate very much to hear about all possible omissions or mistakes by e-mail sgabaimov@gmail.com to the purpose of future corrections. “Needless to say the computer, as a text editing system, should be blamed for all the errors in the book.” (Dietrich Stauffer, in Stauffer, D., Aharony, A.: *Introduction to Percolation Theory*, 2nd ed. Taylor & Francis, London (1994), rephrased).

This book is based on the course of lectures taught by the author for 5 years at the Department of Theoretical Physics of Moscow Institute of Physics and Technology. The first two chapters represent prerequisites. Statistical physics is often considered to be at the top of theoretical disciplines of a student's curriculum and requires the knowledge of previously studied theoretical mechanics and quantum mechanics. This often prohibits the reader not acquainted with these disciplines to study the applicability of statistical physics to complex phenomena.

However, several years of lecturing statistical physics convinced the author that what is truly required to understand the formalism of phase transitions is the discussion of a limited set of concepts. Chapter 2 presents an attempt to reduce the theoretical formalism of statistical physics to a minimum required to understand further chapters. Therefore, as a prerequisite for this monograph we consider only general physics but not theoretical, quantum, or statistical mechanics. It is our belief that Chap. 2 will be sufficient for the reader, not acquainted earlier with theoretical physics, to understand the following chapters.

The completion of this book has left me indebted to many. I am most grateful to Dr. Yury Belousov, Head of the Department of Theoretical Physics at Moscow Institute of Physics and Technology, for his invaluable support and help in the creation of the monograph and course; and also to my colleagues at the Department of Theoretical Physics for fruitful discussions, especially to Dr. Ilya Polishchuk and Dr. Andrey Mikheyenkov. I am most grateful to Dr. Zafer Gürdal, Director of Advanced Structures, Processes and Engineered Materials Center, Skolkovo Institute of Science and Technology, for his support of the monograph and of the course that I am lecturing at ASPeM. I would like to express my warmest gratitude to Dr. Joseph Cusumano, Department of Engineering Science and Mechanics, Penn State University, for his invaluable support and collaboration in the research of damage phenomena. I am also thankful to Dr. Christopher Coughlin, Springer, for his inestimable support and help in the publication of the monograph.

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