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Understanding Complex Systems

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UCS will publish monographs, lecture notes and selected edited contributions aimed at communicating new findings to a large multidisciplinary audience.

Xingjian Jing • Ziqiang Lang

Frequency Domain Analysis and Design of Nonlinear Systems based on Volterra Series Expansion

A Parametric Characteristic Approach

 Springer

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To the authors' families.

Preface

Nonlinearities are ubiquitous and often incur twofold influence, which could be a source of troubles bringing uncertainty, inaccuracy, instability or even disaster in practice, and might also be a superior and beneficial factor for system performance improvement, energy cost reduction, safety maintenance or health monitoring, etc. Therefore, analysis and design of nonlinear systems are important and inevitable issues in both theoretical study and practical applications.

Several methods are available in the literature to this aim including perturbation method, averaging method and harmonic balance method, etc. Nonlinear analysis can also be conducted in the frequency domain based on the Volterra series theory. The latter is a very useful tool with some special and beneficial features to tackle nonlinear problems. It is known that there is a considerably large class of nonlinear systems which allow a Volterra series expansion. Based on the Volterra series, the generalized frequency response function (GFRF) was defined as a multi-variate Fourier transform of the Volterra kernels in the 1950s. This presents a fundamental basis and therefore initiates a totally new theory or area for nonlinear analysis and design in the frequency domain.

The frequency-domain nonlinear analysis theory and methods, based on the Volterra series approach, are observed with a faster development starting from the late 1980s or the early 1990s. Recursive algorithms for computation of the GFRFs for a given parametric nonlinear autoregressive with exogenous input (NARX) model or a given nonlinear differential equation (NDE) model are developed, and output frequency response of nonlinear systems and its properties are investigated accordingly. The area is becoming even more active in recent years. Much more efforts and progress can be seen in the development of application-oriented theory and methods based on the GFRF concept. These include the concepts of nonlinear output spectrum (or output frequency response function) and nonlinear output frequency response function, parametric characteristic analysis, energy transfer properties and various applications in vibration control by exploring nonlinear benefits, fault detection, modelling and identification, data analysis and interpretation, etc.

This book is a systematic summary of some new advances in this area mainly done by the authors in the past years starting from when the first author pursued his Ph.D. degree in the University of Sheffield in 2005. The main results are tried to be formulated uniformly with a parametric characteristic approach, which provides a convenient and novel insight into the nonlinear influence on system output response in terms of characteristic parameters and thus can facilitate nonlinear analysis and design in the frequency domain. The book starts with a brief introduction to the background of nonlinear analysis in the frequency domain, followed by the recursive algorithms for computation of GFRFs for different parametric models, and nonlinear output frequency properties. Thereafter the parametric characteristic analysis method is introduced, which leads to new understanding and formulation of the GFRFs, new concepts about nonlinear output spectrum and new methods for nonlinear analysis and design, etc. Based on the parametric characteristic approach, nonlinear influence in the frequency domain can be investigated with a novel insight, i.e. alternating series, which is followed by some application results in vibration control. Magnitude bounds of frequency response functions of nonlinear systems can also be studied with a parametric characteristic approach, which results in novel parametric convergence criteria for any given parametric nonlinear model whose input–output relationship allows a convergent Volterra series expansion. Although very important and fundamental, these results are summarized at the end of this book.

This book targets those readers (especially Ph.D. students and research staff) who are working in the areas related to nonlinear analysis and design, nonlinear signal processing, nonlinear system identification, nonlinear vibration control and so on. It particularly serves as a good reference for those who are studying frequency-domain methods for nonlinear systems.

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ZQ Lang

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