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Formal Algorithmic Elimination for PDEs

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Preface

This monograph originates in my habilitation treatise. The theme is to understand the correspondence between systems of partial differential equations and their analytic solutions from a formal viewpoint. We consider on the one hand the problem of determining the set of analytic solutions to such a system and on the other hand that of finding differential equations whose set of solutions coincides with a given set of analytic functions. Insight into one of these problems may advance the understanding of the other one. In particular, as shown in this monograph, symbolic solving of partial differential equations profits from the study of the implicitization problem for certain parametrized sets of analytic functions.

Two algorithms are fundamental for this work. Janet's and Thomas' algorithms transform a given system of linear or nonlinear partial differential equations into an equivalent form allowing to determine all formal power series solutions in a straightforward way. Moreover, these effective differential algebra techniques solve certain elimination problems when applied in an appropriate way. The first part of this monograph discusses these methods and related constructions in detail. Efficient implementations of both algorithms in the computer algebra system Maple have been developed by the author of this monograph and his colleagues.

The second part of this monograph addresses the problem of finding an implicit description in terms of differential equations of a set of analytic functions which is given by a parametrization of a certain kind. Effective methods of different generality are developed to solve the differential elimination problems that arise in this context. As a prominent application of these results it is demonstrated how some family of exact solutions of the Navier-Stokes equations can be computed.

I would like to thank several persons for many fruitful discussions about algebra, about PDEs, and their connection, and for supporting my work. In particular, I would like to express my gratitude to Prof. Dr. W. Plesken, Prof. Dr. J. Bemelmans, Prof. Dr. C. Melcher, Prof. Dr. S. Walcher at RWTH Aachen and Priv.-Doz. Dr. M. Barakat at TU Kaiserslautern. Thanks go to Prof. Dr. V. P. Gerdt (JINR Dubna) and Dr. A. Quadrat (Inria Saclay), not only because I have been enjoying our collaborations, but also for reading parts of a first version of this monograph.

In the context of work on Thomas' algorithm, I am grateful to my colleagues Dr. T. Bächler and Dr. M. Lange-Hegermann for their help. Moreover, we profited very much from discussions with Prof. Dr. D. Wang (Paris 6), Prof. Dr. F. Boulier, and Dr. F. Lemaire (both at Université de Lille) on this topic.

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Plymouth, England *Daniel Robertz*
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