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Index Matrices: Towards an Augmented Matrix Calculus

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Preface

Let us start with an apparently strange question to the reader. Suppose that we know what a matrix is but we are not familiar with the matrix calculus, or cannot remember how various matrix operations are defined. Suppose that we have two matrices

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{vmatrix}$$

and ask quite a natural question on what their sum would be.

Clearly, since the first is a 2×3 matrix and the second is 3×2 matrix, then this question is ill-posed in the well-known classic matrix calculus.

However, apart from that obvious fact, we cannot neglect the fact that the above question can be asked by many people and maybe some reasonable answer could be given.

Some 30 years ago, when I was working on many problems involving various aspects of matrix calculus, mostly in areas that could be described as human centric or human centered, a crucial element had been the human being as a key element of the reasoning or decision-making process. Needless to say that in a vast majority of real-world problems the humans involved are very rarely experts in, for instance, mathematics in general and matrix calculus in particular.

Therefore, I started to think how we could introduce some possibly intuitive and natural changes to the very definitions and elements of matrix calculus, notably the matrices themselves and operations on them to obtain some possibly well-justified results.

So, for instance, in the above ill-posed matrix addition examples in which the matrices involved should be of the same size, that is, 3×3 to avoid truncation of any rows from the second matrix that could result in a substantial loss of information, one could imagine that the following possible extensions could be possible:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{vmatrix} \approx \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 10 & 11 & 0 \\ 12 & 13 & 0 \\ 14 & 15 & 0 \end{vmatrix} = \begin{vmatrix} 11 & 13 & 3 \\ 16 & 18 & 6 \\ 14 & 15 & 0 \end{vmatrix}$$

or

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{vmatrix} \approx \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 10 & 11 & 0 \\ 12 & 13 & 0 \\ 14 & 15 & 0 \end{vmatrix} = \begin{vmatrix} 11 & 13 & 3 \\ 12 & 13 & 0 \\ 18 & 20 & 0 \end{vmatrix}$$

or

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{vmatrix} \approx \begin{vmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 10 & 11 & 0 \\ 12 & 13 & 0 \\ 14 & 15 & 0 \end{vmatrix} = \begin{vmatrix} 10 & 11 & 0 \\ 13 & 15 & 3 \\ 18 & 20 & 6 \end{vmatrix}$$

or …

Clearly, there are nine possible ways or setting such slightly changed, or augmented, formulations of our source ill-posed question which would replace it by a well-posed formulation (question) that is apparently the closest in terms of changes.

Of course, the very essence of the matrix additions as given above is not exactly the same of our source, ill-posed, matrix addition problem, and that is why the symbol "$\approx$" was used which stands for more or less equal, similar, etc. Clearly, this should be meant in a proper way due to the fact that we replace an ill-posed matrix calculus problem by a "similarly looking" well-posed one.

To formalize the above way or reasoning, and to devise some plausible means for handling such problems, I developed first the concept of a so-called "index matrix", and then its corresponding augmented matrix calculus that would make it possible to implement those ideas in a plausible and mathematically correct way.

It turned out later that the new concept, properties, operations, etc., have proved to be extremely useful for solving a multitude of problems in many areas of science and technology in which mathematical modeling, notably based on broadly perceived matrix calculus plays a crucial role.

In particular, I have found that the concept of an index matrix has proved to be very useful in the area of generalized nets, an extension of the Petri nets, I introduced some three decades ago, and which had enjoyed since then a wide popularity as an effective an efficient tool for modeling and solution of a wide array of problems involving systems that can be viewed as being of a discrete event system type. Moreover, I have found a similar importance of the concept of an index matrix in the context of intuitionistic fuzzy sets which I introduced some three decades ago and which, again, had enjoyed since then a booming popularity.

I will therefore present in this book in a comprehensive form the very concept of an index matrix and its related augmented matrix calculus, and will mostly illustrate my exposition with examples related to the generalized nets and intuitionistic fuzzy sets though one should remember that these are just examples of an extremely wide array of possible application areas.

This book contains the basic results of mine over index matrices and some of the open problems concerning them. I will be very glad if the book succeeds in provoking scientific interest and stimulating other fellow researchers to start working in this area.

I am very thankful to my Ph.D. students Velin Andonov, Peter Hadjistoykov, Evgeniy Marinov, Peter Vassilev, and my daughter Vassia Atanassova, who motivated me to prepare the present book and corrected the text and to my coauthors for papers in which the theory of index matrices has been developed: Anthony Shannon (Australia), Eulalia Szmidt and Janusz Kacprzyk (Poland), Veselina Bureva, Deyan Mavrov, Evdokia Sotirova, and Sotir Sotirov (Bulgaria).

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Sofia, Bulgaria, June 2014 Krassimir T. Atanassov

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