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Polynomial Chaos Methods for Hyperbolic Partial Differential Equations

Numerical Techniques for Fluid Dynamics Problems in the Presence of Uncertainties

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Preface

Uncertainty quantification in computational physics is a broad research field that has spurred increasing interest in the last two decades, partly due to the growth of computer power. The objective of this textbook is the analysis and design of numerical techniques for solving equations representing conservation laws subject to uncertainty. In particular, the focus is on stochastic Galerkin methods that require non-trivial development of new numerical solvers for hyperbolic and mixed type problems. There are already textbooks covering the stochastic Galerkin and other polynomial chaos methods from a general perspective, cf. [1–3]: this textbook is more specialized in its scope. To enhance understanding of the material presented, we provide exercises and code scripts and building blocks that can be extended to new problem settings.

The interest in stochastic Galerkin methods has burgeoned because of the availability of ever more powerful computers that can handle the computational cost inherent to large system implementations. Moreover, these methods have positive numerical properties that make them attractive for handling complex situations. Specifically, the mathematical formulation leads to systems of equations that resemble the original conservation laws, allowing us to make extensive use of available numerical analysis tools and techniques. At the same time, the stochastic Galerkin method is an attractive alternative for complex problems involving partial differential equations and multiple uncertain variables (herein referred to as stochastic dimensions).

Chapters 1–3 introduce and give a brief overview of the basic concepts of uncertainty quantification and the stochastic Galerkin method. Chapter 4 is devoted to spatial discretization methods for conservation laws under uncertainty. In particular, we introduce the so-called SBP-SAT finite difference technique based on summation-by-parts operators (SBP) and weak boundary conditions using simultaneous approximation terms (SAT). The SBP-SAT schemes allow for the design of stable high-order accurate schemes. Summation by parts is the discrete equivalent of integration by parts and the matrix operators that are presented lead to energy estimates that, in turn, lead to provable stability in combination with the SAT terms. The semidiscrete stability follows naturally from the continuous analysis of well-posedness which provides the boundary conditions in the SBP-SAT technique. Chapters 5–9 present in-depth analysis of linear and nonlinear stochastic Galerkin conservation laws, complemented by exercises and scripts. We provide the reader with computer codes for solving the advection-diffusion equation and the inviscid Burgers' equation with the stochastic Galerkin method. These codes can also be used as templates for extension to more complex problems.

This textbook is intended for an audience with some prior knowledge of uncertainty quantification. Basic concepts of probability theory, statistics and numerical analysis are also assumed to be familiar to the reader. For a more general exposition and further details on the basic concepts, we refer to the existing literature in the field.

This textbook has benefited from numerous collaborations and discussions with Alireza Doostan (who co-authored the material contained in Chap. 5), Antony Jameson, Xiangyu Hu, Rémi Abgrall and Paul Constantine. We would like to thank Margot Gerritsen for constructive feedback and suggestions for improvement. Financial support was partially provided by KAUST under the Stanford/KAUST Academic Excellence Alliance (AEA) collaboration (UDGIA Award 48803). Gianluca Iaccarino wishes to thank the *Borrister crew* for support in completing the final revision of the text.

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Contents

Acronyms

CFL	Courant-Friedrichs-Lewy
ENO	Essentially Non-Oscillatory Scheme
gPC	Generalized polynomial chaos
HLL	Harten-Lax; van Leer
IBVP	Initial-boundary value problem
KL	Karhunen-Loève
ME-gPC	Multi-element generalized polynomial chaos
MUSCL	Monotone upstream-centered scheme for conservation laws
MW	Multiwavelet
ODE	Ordinary differential equation
PC	Polynomial chaos
PDE	Partial differential equation
SAT	Simultaneous approximation term
SBP	Summation by parts
TVD	Total variation diminishing
UQ	Uncertainty quantification