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Variational Inequalities and Frictional Contact Problems

 Springer

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In the memory of my father, Dumitru Stoleru

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