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Series Editors

Professor Richard Needs

Cavendish Laboratory
JJ Thomson Avenue
Cambridge CB3 0HE, UK
rn11@cam.ac.uk

Professor William T. Rhodes

Department of Computer and Electrical Engineering and Computer Science
Imaging Science and Technology Center
Florida Atlantic University
777 Glades Road SE, Room 456
Boca Raton, FL 33431, USA
wrhodes@fau.edu

Professor Susan Scott

Department of Quantum Science
Australian National University
Science Road
Acton 0200, Australia
susan.scott@anu.edu.au

Professor H. Eugene Stanley

Center for Polymer Studies Department of Physics
Boston University
590 Commonwealth Avenue, Room 204B
Boston, MA 02215, USA
hes@bu.edu

Professor Martin Stutzmann

Walter Schottky Institut
TU München
85748 Garching, Germany
stutz@wsi.tu-muenchen.de

Michel Rieutord

Fluid Dynamics

An Introduction

Springer

Michel Rieutord
Institut de Recherche en Astrophysique et Planétologie
Université Paul Sabatier
Toulouse
France

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Preface

The idea that guided the first French edition of the present book was to give to newcomers in Fluid Dynamics a presentation of the field that was anchored in Physics rather than in Applied Mathematics as it had been the case so often in the past. Presently, however, connections with Physics are getting stronger and this is fortunate. Indeed, Physics is, etymologically, the science of Nature and fluids occupy a major place in Nature. They are everywhere around us and their motion (their mechanics) influences our everyday life, at least through the weather. Any physicist can hardly escape being fascinated by the sight of some remarkable fluid flows like breaking waves or the gently travelling smoke ring.

The connection between Fluid Mechanics and Applied Mathematics is certainly understandable by the very small number of equations that control a fluid flow. This is fascinating for an applied mathematician, especially if keen on the theory of partial differential equations. Actually, a few decades ago, expertise in asymptotic expansions, singular perturbations, and other mathematical technics was a necessary condition to make progress in the theory of fluid flows. But the pressure of maths has certainly lessened in the recent times because of the strong (exponential) growth of numerical simulations. It is now easier to experiment numerically a fluid flow and get a detailed description of the solutions of Navier–Stokes equation. Interpretation of the results may challenge the intuition of the physicist rather than the skill of the mathematician. But even in the pioneering times, when theoretical investigations of fluid flows were at the strength of the pencil, famous physicists like Newton, Maxwell, Kelvin, Rayleigh, Heisenberg, Landau, Chandrasekhar, and others made essential contributions to the field of Fluid Dynamics. As noted by Heisenberg himself, the theory of turbulence awaits to be written, and this is still the case.

The present book is based on the lectures I delivered at Paul Sabatier University in Toulouse during the last two decades. It is intended to beginners in the field and aims at providing them with the necessary basis that will allow them to attack most of Fluid Dynamics questions. I have tried, as much as possible, to illustrate the concepts with examples taken in natural sciences, often in Astrophysics, which is my playground. Some exercises are offered at the end of each chapter. The

reader may thus check his/her understanding of the text. Some of the exercises are also meant to extend the subject in a different way. In that respect, I also give some references for further reading. As far as maths are concerned, the last chapter proposes some brief reminders or introduction to the mathematical tools that are used in the text. With the solutions of the exercises, the book should be self-contained.

As far as teaching is concerned, the first four chapters constitute the bulk of a Fluid Mechanics introduction to third year students. The four following chapters were typically taught to fourth year students, while part of the last ones are currently taught to students about to start a Ph.D. As the reader will note, some sections are tagged with ♠. They can be skipped at first reading and present other illustrations of the subject of the chapter.

Ending this short preface, I would like to thank the many colleagues who have, by various means, contributed to the achievement that a book writing represents. I would like to specially thank Alain Vincent and Hervé Willaime who provided me with original data of turbulent flows. I have much benefitted from the remarks of Arnaud Antkowiak, Pierre-Louis Blelly, Boris Dintrans, Katia Ferrière and Thierry Roudier. They helped me very much at improving various parts of the work. I cannot forget that this adventure of writing started, thanks to the support and help of José-Philippe Pérez. I know that my wife Geneviève and my children Clément and Sylvain will forgive me for the many hours spent outside the real world. The realization of the present book owes much to the kind support of Dr. Ramon Khanna of Springer; I thank him very much for his faith in the project. Finally, I should thank the many students who attended the performance written below, their questions were always beneficial, their enthusiasm always stimulating and their fear challenging for the teacher.

Toulouse, France Michel Rieutord
May 2014

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