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Random Sets and Random Fuzzy Sets as Ill-Perceived Random Variables

An Introduction for Ph.D. Students
and Practitioners

 Springer

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Preface

Several books written by prominent authors during the last 40 years have covered different aspects of random sets rather independently of one another. In most cases, random sets are considered as precise representations of random complex entities whose outcomes were naturally described by sets of elements. Independently, the theory of belief functions uses a similar formal setting to describe an extension of subjective probabilities that accommodates incomplete information. Besides, the recent literature contains many papers about fuzzy random variables, that often adopt the first point of view. This book is intended to focus on the use of random sets and fuzzy random variables as natural representations of ill-observed variables or partially known conditional probabilities.

The content of this book is based on several joint chapters written by the authors, enlarged and completed with examples and exercises. It deals with the use of random sets and fuzzy random variables from an epistemic point of view. This is closely connected to the possibilistic interpretation of fuzzy sets, suggested by L. A. Zadeh in 1978. Within this context, the relation between possibility measures and families of nested confidence intervals, and their relation with cuts of fuzzy sets was independently studied by Inés Couso and Luciano Sánchez, and Didier Dubois and colleagues during the last decade. Later on, a joint study about the combination of possibility and probability measures, carried out in the context of the Ph.D. thesis of Cédric Baudrit, could shed light on various related issues, especially connecting Dempster upper and lower probabilities to fuzzy random variables, and questioning the relevance of classical approaches proposed to evaluate statistical parameters in this setting.

The terms “ontic” and “epistemic” fuzzy sets were suggested by D. Dubois and H. Prade in order to distinguish between complex set-valued entities and the representation of partial knowledge about point-valued entities. This distinction naturally leads to three different interpretations of fuzzy random variables, one of them related to the ontic interpretation of fuzzy sets, and the remaining two related to the epistemic one. An initial joint work in the context of the Ph.D. thesis by Cédric Baudrit served to deepen the study of this categorization. Long discussions between the authors about possible meanings of sets and fuzzy sets, led them to

propose three different interpretations of fuzzy random variables, that are present in this book.

The authors are indebted to other colleagues for numerous discussions. Serafín Moral and Gert de Cooman were kind enough to review the work of Inés Couso in early stages of her research activity, and to suggest interesting ideas. Some discussions with them about objective and subjective views of probabilities and previsions were very enriching. Debates with María Ángeles Gil, Ana Colubi, and Reinhard Viertl about the nature of fuzzy set-valued data were also fruitful. Part of those debates took place at the first Workshop on Harnessing the Information Contained in Low Quality Data that took place in Mieres (Spain) in 2012, and are reported in a Special Issue of the *International Journal of Approximate Reasoning*, edited by Inés Couso and Luciano Sánchez. During the final stages of her Ph.D. studies, Inés Couso had the opportunity to speak with Peter Walley about some of the aspects about information preserved by nonconvex sets of probabilities induced by random sets and the loss of relevant information about certain parameters involved in the convexification of such a family of probabilities. They also discussed (along with Serafín Moral) about different aspects of the notions of conditioning and independence in the context of imprecise probabilities.

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