

Springer Proceedings in Mathematics & Statistics

Volume 94

For further volumes:

<http://www.springer.com/series/10533>

Springer Proceedings in Mathematics & Statistics

This book series features volumes composed of select contributions from workshops and conferences in all areas of current research in mathematics and statistics, including OR and optimization. In addition to an overall evaluation of the interest, scientific quality, and timeliness of each proposal at the hands of the publisher, individual contributions are all refereed to the high quality standards of leading journals in the field. Thus, this series provides the research community with well-edited, authoritative reports on developments in the most exciting areas of mathematical and statistical research today.

Ferenc Hartung • Mihály Pituk
Editors

Recent Advances in Delay Differential and Difference Equations

 Springer

Editors

Ferenc Hartung
Department of Mathematics
University of Pannonia
Veszprém, Hungary

Mihály Pituk
Department of Mathematics
University of Pannonia
Veszprém, Hungary

ISSN 2194-1009

ISBN 978-3-319-08250-9

DOI 10.1007/978-3-319-08251-6

Springer Cham Heidelberg New York Dordrecht London

ISSN 2194-1017 (electronic)

ISBN 978-3-319-08251-6 (eBook)

Library of Congress Control Number: 2014946773

Mathematics Subject Classification (2010): 34C10, 34D05, 34F05, 34K05, 34K20, 35A05, 35R10, 37B20, 37B55, 37H10, 39A10, 93C05, 93C10, 93E15

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

We would like to recommend the readers the 12 research papers on differential and difference equations in this volume. Differential and difference equations are to be understood in the broad sense. They include discrete and continuous dynamical systems, stochastic differential and difference equations, and numerical simulations of the solutions and applications.

The papers are related to the research presented by the corresponding authors at the “International Conference on Delay Differential and Difference Equations and Applications, July 15–19, 2013, Balatonfüred, Hungary” organized by the Department of Mathematics, Faculty of Information Technology of the University of Pannonia, Veszprém, Hungary. This conference was dedicated to the 70th birthday of our colleague, Professor István Győri. He has been working at the University of Pannonia, Hungary, as a full-professor of mathematics since 1993. He was the head of the Department of Mathematics between 1993 and 2009, and in the period 1995–1998 he served as the president of the university.

István Győri has published more than 160 scientific papers on differential and difference equations including his monograph on oscillation theory written jointly with Professor Gerry Ladas (*Oscillation Theory for Delay Differential Equations with Applications*, Oxford University Press, Oxford, 1991). In addition, he has more than 50 papers in medical and informatics applications. His open problems and papers have motivated further research, and more than 2,800 citations to his papers can be counted in the literature. He published papers together with more than 80 coauthors, and he has been a supervisor of several Ph.D. dissertations.

István Győri acts as a member of the editorial boards of more than ten international scientific journals, and he has been an invited lecturer and a member of the scientific and organizing committees of numerous international conferences. Since 2004 he has been a member of the boards of directors of the International Society of Difference Equations.

Finally, we remark that each paper in this volume has been carefully reviewed. We express our sincere thanks to the referees for their service to help our editorial task.

Veszprém, Hungary
March 2014

Ferenc Hartung
Mihály Pituk

Contents

1	On Necessary and Sufficient Conditions for Preserving Convergence Rates to Equilibrium in Deterministically and Stochastically Perturbed Differential Equations with Regularly Varying Nonlinearity	1
	John A.D. Appleby and Denis D. Patterson	
1.1	Introduction.....	1
1.2	Preliminaries.....	9
	1.2.1 Notation and Properties of Regularly Varying Functions ..	9
1.3	Asymptotic Behaviour for Ordinary Differential Equations with Internal Perturbations.....	10
	1.3.1 Main Result and Discussion.....	10
	1.3.2 Application of Theorem 1.1 to (1.1) and (1.2).....	11
1.4	Main Results for Perturbed ODE.....	13
1.5	Main Results for SDEs.....	17
	1.5.1 Asymptotic Decay Rates of Solutions of (1.2).....	17
	1.5.2 Characterisation of Preserved Decay Rate in Terms of an Upper Class Condition.....	21
	1.5.3 The Scaled Increments of X	29
1.6	Examples.....	31
1.7	Simulations.....	39
1.8	Proof of Theorem 1.1.....	41
	1.8.1 Idea and Outline of the Proof.....	41
	1.8.2 Statement and Proofs of Technical Results.....	43
1.9	Proofs from Sect. 1.4.....	55
1.10	Proofs from Sect. 1.5.....	60
1.11	Proof of Theorems 1.14 and 1.15.....	66
1.12	Proofs from Examples Section.....	77
	References.....	84

2	Comparison Theorems for Second-Order Functional Differential Equations	87
	Zuzana Došlá and Mauro Marini	
2.1	Introduction.....	87
2.2	Preliminaries.....	88
2.3	Intermediate Solutions	93
2.4	Applications	98
2.5	The Coexistence of Nonoscillatory Solutions.....	100
2.6	Open Problems	102
	References.....	102
3	Analysis of Qualitative Dynamic Properties of Positive Polynomial Systems Using Transformations	105
	Katalin M. Hangos and Gábor Szederkényi	
3.1	Introduction.....	106
3.2	Quasi-Polynomial (QP) Systems	107
3.2.1	The ODE Form.....	107
3.2.2	Quasi-Monomial Transformation and the Lotka-Volterra Canonical Form	107
3.2.3	The Time-Rescaling Transformation.....	109
3.2.4	Stability Condition for QP Systems.....	110
3.3	Chemical Reaction Networks with Mass Action Law	111
3.3.1	Formal Description	111
3.3.2	MAL-CRN Structural Stability.....	113
3.3.3	Linear CRN Systems	114
3.4	Transforming LV Models to a Linear MAL-CRN Form	115
3.4.1	The Translated X-Factorable Transformation	115
3.4.2	Constructing a Dynamically Similar Linear CRN Form.....	116
3.4.3	Structural Stability Analysis.....	117
3.5	Conclusion and Future Work	118
	References.....	118
4	Almost Oscillatory Solutions of Second Order Difference Equations of Neutral Type	121
	Robert Jankowski and Ewa Schmeidel	
4.1	Introduction.....	121
4.2	Main Results.....	123
	References.....	129
5	Uniform Weak Disconjugacy and Principal Solutions for Linear Hamiltonian Systems	131
	Russell Johnson, Sylvia Novo, Carmen Núñez, and Rafael Obaya	
5.1	Introduction and Preliminaries.....	131
5.2	Uniform Weak Disconjugacy and Principal Solutions	135

5.3	Disconjugacy, Uniform Weak Disconjugacy, and Weak Disconjugacy.....	144
5.4	General Properties of the Principal Functions	152
	References.....	158
6	Stability Criteria for Delay Differential Equations	161
	Beáta Krasznai	
6.1	Introduction.....	161
6.2	Summary of Known Results	163
6.3	Stability Criteria.....	166
	References.....	170
7	Analyticity of Solutions of Differential Equations with a Threshold Delay	173
	Tibor Krisztin	
7.1	Introduction.....	173
7.2	The Result	174
	References.....	179
8	Application of Advanced Integrodifferential Equations in Insurance Mathematics and Process Engineering	181
	Éva Orbán-Mihálykó and Csaba Mihálykó	
8.1	Introduction.....	181
8.2	The Integral Equation for $m_\beta(x)$	184
8.3	An Integrodifferential Equation for LODE-Type Inter-Arrival Time Distribution	186
8.4	The Lundberg Fundamental Equation of the Model.....	190
8.5	An Analytical Solution of the Integrodifferential Equation	192
8.6	Summary.....	194
	References.....	195
9	Stability and Control of Systems with Propagation.....	197
	Vladimir Räsvan	
9.1	Introduction and Motivation	198
9.2	A Benchmark Dynamics: The Overhead Crane and its Mathematical Model	200
9.3	The Basic Theory for System (9.11)	203
9.4	The Energy Identity and the Feedback Stabilization	206
9.5	Asymptotic Stability in a Limit Case.....	208
9.6	On the Basic Theory and Asymptotic Stability for the Closed-Loop System	212
9.7	Some Conclusions and Open Problems	216
	References.....	216

10 Discrete Itô Formula for Delay Stochastic Difference Equations with Multiple Noises 219
 Alexandra Rodkina

10.1 Introduction 219

10.2 Preliminaries 221

10.3 Itô Formula 222

10.4 Stability 226

10.5 Instability 229

10.6 Example 231

References 231

11 On Semilinear Hyperbolic Functional Equations with State-Dependent Delays 233
 László Simon

11.1 Introduction 233

11.2 Existence in $(0, T)$ 234

11.3 Examples 242

11.4 Solutions in $(0, \infty)$ 243

References 250

12 A Fast Parallel Algorithm for Delay Partial Differential Equations Modeling the Cell Cycle in Cell Lines Derived from Human Tumors 251
 Barbara Zubik-Kowal

12.1 Introduction 251

12.2 Model Equations with Time Delay Terms 253

12.3 Parallel Algorithm 255

12.4 Concluding Remarks 258

References 259

Index 261

List of Contributors

John A.D. Appleby

Edgeworth Centre for Financial Mathematics, School of Mathematical Sciences,
Dublin City University, Glasnevin, Dublin, Ireland

Zuzana Došlá

Department of Mathematics and Statistics, Masaryk University, Brno, Czech
Republic

Katalin M. Hangos

Department of Electrical Engineering and Information Systems, University of
Pannonia, Veszprém, Hungary
Computer and Automation Research Institute, Budapest, Hungary

Robert Jankowski

Technical University of Lodz, University of Bialystok, Bialystok, Poland

Russell Johnson

Dipartimento di Matematica e Informatica, Università di Firenze, Firenze, Italy

Beáta Krasznai

Department of Mathematics, University of Pannonia, Veszprém, Hungary

Tibor Krisztin

Bolyai Institute, MTA-SZTE Analysis and Stochastic Research Group, University
of Szeged, Szeged, Hungary

Mauro Marini

Department of Mathematics and Informatics “Ulisse Dini”, University of Florence,
Florence, Italy

Csaba Mihálykó

Department of Mathematics, University of Pannonia, Veszprém, Hungary

Sylvia Novo

Departamento de Matemática Aplicada, Escuela de Ingenierías Industriales, Universidad de Valladolid, Valladolid, Spain

Carmen Núñez

Departamento de Matemática Aplicada, Escuela de Ingenierías Industriales, Universidad de Valladolid, Valladolid, Spain

Rafael Obaya

Departamento de Matemática Aplicada, Escuela de Ingenierías Industriales, Universidad de Valladolid, Valladolid, Spain

Éva Orbán-Mihálykó

Department of Mathematics, University of Pannonia, Veszprém, Hungary

Denis D. Patterson

School of Mathematical Sciences, Dublin City University, Glasnevin, Dublin, Ireland

Vladimir Răsvan

Department of Automation, University of Craiova, Electronics and Mechatronics, Craiova, Romania

Alexandra Rodkina

Department of Mathematics, University of the West Indies, Kingston, Jamaica

Ewa Schmeidel

University of Białystok, Białystok, Poland

László Simon

Department of Applied Analysis and Computational Mathematics, Eötvös Loránd University, Budapest, Hungary

Gábor Szederkényi

Peter Pázmány Catholic University, Budapest, Hungary

Barbara Zubik-Kowal

Department of Mathematics, Boise State University, Boise, ID, USA

Acronyms

BVP	Boundary value problem
CRN	Chemical reaction network
FDE	Functional differential equation
LMI	Linear matrix inequality
LODE	Linear ordinary differential equation
LV	Lotka-Volterra
MAL-CRN	Chemical reaction network with mass action law
NFDE	Neutral functional differential equation
ODE	Ordinary differential equation
PD	Proportional derivative
PDE	Partial differential equation
QM	Quasi-monomial
QP	Quasi-polynomial
SDE	Stochastic differential equation