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Stability of the Turnpike Phenomenon in Discrete-Time Optimal Control Problems

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Preface

The monograph is devoted to the study of the structure of approximate solutions of nonconvex (nonconcave) discrete-time optimal control problems. It contains new results on properties of approximate solutions which are independent of the length of the interval, for all sufficiently large intervals. These results deal with the so-called turnpike property of optimal control problems. The term was first coined by P. Samuelson in 1948 when he showed that an efficient expanding economy would spend most of the time in the vicinity of a balanced equilibrium path (also called a von Neumann path). To have the turnpike property means, roughly speaking, that the approximate solutions of the problems are determined mainly by the objective function and are essentially independent of the choice of interval and endpoint conditions, except in regions close to the endpoints. Now it is well-known that the turnpike property is a general phenomenon which holds for large classes of variational and optimal control problems. Using the Baire category (generic) approach, it was shown that the turnpike property holds for a generic (typical) variational problem [45] and for a generic optimal control problem [56]. According to the generic approach we say that a property holds for a generic (typical) element of a complete metric space (or the property holds generically) if the set of all elements of the metric space possessing this property contains a G? everywhere dense subset of the metric space which is a countable intersection of open everywhere dense sets. In [55] we were interested in individual (non-generic) turnpike results and in sufficient and necessary conditions for the turnpike phenomenon in the calculus of variations. In our recent research [46-51, 54] we were are also interested in individual turnpike results but for discrete-time optimal control problems which, in particular, describe a general model of economic dynamics. For these problems we established the turnpike property for approximate solutions with a singleton-turnpike and studied the stability of the turnpike phenomenon under small perturbations of objective functions.

In this book we continue to study the discrete-time optimal control problems considered in [46-51, 54]. Some results of these works are discussed in Chap. 1. In Chaps. 2 and 3 we show the stability of the turnpike phenomenon under small perturbations of objective functions and under small perturbations of control maps. The optimal control problems without discounting are studied in Chap. 2 while the discount case is considered in Chap. 3. In Chap. 4 we establish the turnpike property and its stability for discrete-time problems with nonsingleton-turnpikes. Note that the stability of the turnpike property is crucial in practice. One reason is that in practice we deal with a problem which consists of a perturbation of the problem we wish to consider. Another reason is that the computations introduce numerical errors.

Rishon LeZion Alexander J. Zaslavski
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List of Symbols