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Physics Editorial Department I
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Andreas Schmitt

Introduction to Superfluidity

Field-theoretical Approach and Applications

 Springer

Andreas Schmitt
Institut für Theoretische Physik
Technische Universität Wien
Wien
Austria

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Preface

This course is about the theory of low-energy and high-energy, non-relativistic and relativistic, bosonic and fermionic superfluidity and superconductivity. Does that sound too much? Well, one important point of the course will be to show that these things are not as diverse as they might seem: the mechanism behind and the basic phenomenological properties of superfluidity are the same whether applied to “ordinary” low-energy superfluids or to more “exotic” superfluids in high-energy physics; non-relativistic and relativistic treatments may look quite different at first sight, but of course the former is only a limit case of the latter; bosonic and fermionic superfluids can be continuously connected in some sense; and, once you understand what a superfluid is, it is very easy to understand what a superconductor is and vice versa.

The motivation for this course arose from my own research in high-energy physics where certain kinds of superfluids and superconductors are predicted in ultra-dense nuclear and quark matter. These are “stellar superfluids”, since they are likely to occur in the interior of compact stars. Working on stellar superfluids, it was natural to learn about more down-to-earth superfluids which are firmly established experimentally. Therefore, this course is interesting for researchers who are in a similar situation like myself, who have some background in high-energy physics and want to learn about superfluidity, explained in a field-theoretical language they are used to. I believe that the course is also insightful for researchers with a background in condensed matter physics who are interested in high-energy applications of their field and a relativistic field-theoretical formalism they usually do not employ. And, most importantly, this course is intended for advanced undergraduate students, graduate students, and researchers who simply want to understand what superfluidity is and what its applications in modern physics are.

Readers unfamiliar with quantum field theory might find some of the chapters challenging, even though I have tried to present most of the calculations in a self-contained way. When this was not possible, I have mentioned suitable references where the necessary elements of field theory are explained. However, not all of the chapters rely on field-theoretical methods. For instance, the course starts with an introduction to superfluid helium that can easily be understood with basic

knowledge of statistical physics and thermodynamics. Most of the chapters that do employ quantum field theory aim at a microscopic description of superfluids, i.e., the degrees of freedom of the theory are the bosons that condense or the fermions that form Cooper pairs. In this sense, the course is in large parts about the fundamental mechanisms behind superfluidity. But, I will emphasize the connection to phenomenology throughout the course and do not want the reader to get lost in technical details. For instance, I will show in a simple setting how a microscopic quantum field theory can be connected to the phenomenological two-fluid model of a superfluid.

Despite the pompous announcement in the first sentence, this is a course that can be taught in about one semester. Therefore, it can only deal with a few selected aspects of superfluidity. This selection has been based on the aim to convey the underlying microscopic physics of superfluidity, on pedagogical considerations, and of course is also, to some extent, a matter of taste. As a result of this subjective selection, there are many important aspects that I will not, or only marginally, discuss, such as vortices in a rotating superfluid, dissipative effects, or observable signatures of stellar superfluids. Literature that can be consulted for such topics and for further reading in general is given at the end of the introduction and throughout the text.

These lecture notes are based on a course that I taught at the Vienna University of Technology in the winter semester 2011/2012 and in the summer semester 2013. I would like to thank all participants for numerous questions and many lively discussions that have improved my understanding of superfluidity. I am grateful to Mark Alford, Karl Landsteiner, S. Kumar Mallavarapu, David Müller, Denis Parganlija, Florian Preis, Anton Rebhan, and Stephan Stetina for many helpful comments and discussions. This work has been supported by the Austrian science foundation FWF under project no. P23536-N16 and by the NewCompStar network, COST Action MP1304.

Vienna, Austria
April 2014

Andreas Schmitt

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