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Input Modeling with Phase-Type Distributions and Markov Models

Theory and Applications

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Preface

Nowadays system analysis of man-made systems, like computer systems, communication networks, and manufacturing plants, and also of natural systems, like biological or social systems, is often model based. To capture the complexity of real systems stochastic discrete event models are used in many application areas. One of the key aspects in building such models is the adequate description of real processes and event streams in a stochastic model. Often simple distributions are not sufficient for this purpose because observed distributions are multimodal and events are correlated.

One class of stochastic models, which allows one to describe multimodal distributions and correlated event times, are Markov processes with marked transitions. Since Markov processes can be analyzed with numerical methods and with stochastic simulation, they are an ideal candidate to describe event times in stochastic models. However, the big disadvantage of using Markov processes instead of simple distributions or stochastic processes, like autoregressive or moving average time series, is the parameterization effort. Usually, the finding of adequate parameters of a Markov model, to capture some observed behavior, is a non-linear optimization problem with many parameters and non-unique representations of a given stochastic distribution or process. This often prohibits the wider use of those models, in particular in stochastic simulation, and is the reason that only fairly simple phase-type distributions can be found in textbooks on stochastic modeling or simulation.

This books summarizes our work on the parameterization of phase-type distributions and Markovian arrival processes, which are the commonly used model types in modeling event streams. To the best of our knowledge it is the first time that the available methods are collected in a textbook. We hope that this helps to support the use of the mentioned models in stochastic modeling and in particular in stochastic simulation.

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Contents

Acronyms

ALS	Alternating least squares approach
APHD	Acyclic phase-type distribution
BMAP	Batch Markovian arrival process
BS	Basic series of an APH
cdf	Cumulative distribution function
CTMC	Continuous time Markov chain
EM	Expectation maximization
HErD	Hyper-Erlang distribution
IPP	Interrupted Poisson process
KPC	Kronecker product composition
MAP	Markovian arrival process
ME	Matrix exponential
ML	Maximum-likelihood
MMAP	Marked Markovian arrival process
MMPP	Markov modulated Poisson process
MRAP	Marked rational arrival process
NNLS	Non-negative least squares approach
PH	Phase-type
PHD	Phase-type distribution
QBD	Quasi-birth-death process
QN	Queueing network
RAP	Rational arrival process

Notation

$\mathbf{0}$	Matrix or vector where every entry is 0
$\mathbb{1}$	(Column) vector where every entry is 1
$APHD(n)$	APHD of order n
$\mathbb{B}$	Set of boolean values
B_i	Number of times a PHD or MAP starts in phase i
C, C^2	(Squared) coefficient of variation
$\mathbf{D}_0$	Matrix of internal transition rates of a PHD or MAP
$\mathbf{D}_1$	Matrix of transition rates generating an event of a MAP
$\mathbf{d}_1$	Exit vector of a PHD
$E[X]$	Expectation of random variable X
$\mathbf{H}_n$	Matrix of factorial moments up to order n
$\mathbf{I}$	Identity matrix
$\lambda(i)$	Transition rate out of state i
$\lambda(i,j)$	Transition rate between the states i and j
$\mathcal{L}(\mathbf{\Theta})$	Likelihood function
$\mathbf{M}$	$= -\mathbf{D}_0^{-1}$ moment matrix of a PHD or MAP, i.e. fundamental matrix of an absorbing Markov chain
$MAP(n)$	MAP of order n
μ_i	i-th moment
$\hat{\mu}_i$	Estimator of the i-th moment
μ_{kl}	Joint moment of order k,l of two consecutive inter-event times
$\hat{\mu}_{kl}$	Estimator of the joint moment of order k,l of two consecutive inter-event times
M_{ij}	Number of transitions from phase i to j with generating an event
n	Order of a PHD or MAP
n_i	i-th normalized moment
N_{ij}	Number of transitions from phase i to j without generating an event
$N(t)$	Number of events of a counting process in the interval $[0,t]$
$\mathbb{N}$	The set of natural numbers
Ω, Ω_I	Partition of the state space and partition group
$\boldsymbol{\pi}$	Initial vector of a PHD or a MAP

$\boldsymbol{\pi}_s$	Stationary vector of a MAP at event generation time points
p_t	Probability distribution of a CTMC at time t
$\mathbf{P}$	Transition probability matrix of an embedded Markov process of a continuous-time Markov-chain
$\mathbf{P}_0$	$= \mathbf{D}_0/\alpha + \mathbf{I}$, matrix of the discrete time Markov-chain used for uniformization
$\mathbf{P}_1$	$= \mathbf{D}_1/\alpha$, matrix of transitions related to events in the discrete time Markov-chain used for uniformization
$\mathbf{P}_s$	$= (-\mathbf{D}_0)^{-1}\mathbf{D}_1$, matrix of the discrete time Markov-chain at event generation time points
$PHD(n)$	PHD of order n
$\mathcal{P}$	Process (describing a real system or an adequate simulation model)
$\mathbf{Q}$	Infinitesimal generator matrix of a continuous-time Markov-chain
ρ_k	Coefficient of autocorrelation at lag k
$\hat{\rho}_k$	Estimator for the coefficient of autocorrelation at lag k
r_i	i-th factorial moment
$\mathbb{R}$	Set of real numbers
$\mathcal{S}$	State space of a CTMC
$\mathcal{S}_T$	Set of transient states of an absorbing CTMC
$\mathcal{S}_A$	Set of absorbing states of an absorbing CTMC
$\mathcal{T}$	Trace, i.e. a sequence of observations usually measured from a real system
$\mathcal{T}^*, \bar{\mathcal{T}}$	Aggregated trace
$\tilde{\mathcal{T}}$	Grouped trace
$\mathbf{V}$	Collector matrix of an aggregation
$VAR[Y]$	Variance of random variable Y
$\mathbf{W}$	Distributor matrix of an aggregation
$X(t)$	Stochastic process
Z_i	Total time spent in phase i of a PHD or MAP before generating an event