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Herbert Edelsbrunner

A Short Course in Computational Geometry and Topology

 Springer

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Preface

This short book has been written with the purpose to bring the subject of *Computational Geometry and Topology* to a wider audience, namely to the scientists who deal with shapes in their work. To gain some focus, we have limited the scope to a small subset of topics, selected to form a coherent and convincing story. This includes fundamental topics and those that are relevant to applications—and we do not see any contradiction in these two requirements.

Coherence and relevance were also the selection guidelines when we developed the course at the Institute of Science and Technology Austria. Indeed, I have taught versions of the material three times: in 2010/2011 as an advanced course with Paul Bendich, in 2012 as a core course with Michael Kerber, and in 2013 as a core course by myself. I owe both, Paul and Michael, for their help in selecting topics and in finding innovative ways to teach the material. At IST Austria, we do not have departments, and core courses are taught to a mixed population of students: mathematicians, computer scientists, biologists, and neuroscientists. Similar to a typical course in the USA, the material is delivered in two lectures of 75 min each per week, and we have arranged the topics such that each section contains the material for a single lecture. A core course at IST Austria lasts for half a semester, which explains the small number of sections.

In conclusion, we recommend the prospective reader to approach this short book like a text in mathematics, and not like a novel. Read it slowly, pay close attention to detail, think about the concepts, and spend time to digest the material.

Klosterneuburg, Austria, 2013

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