

Signals and Communication Technology

For further volumes:
<http://www.springer.com/series/4748>

Frank Nielsen
Editor

Geometric Theory of Information

 Springer

Editor

Frank Nielsen
Sony Computer Science Laboratories Inc
Shinagawa-Ku, Tokyo
Japan

and

Laboratoire d'Informatique (LIX)
Ecole Polytechnique
Palaiseau Cedex
France

ISSN 1860-4862 ISSN 1860-4870 (electronic)
ISBN 978-3-319-05316-5 ISBN 978-3-319-05317-2 (eBook)
DOI 10.1007/978-3-319-05317-2
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014939042

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

*To Audrey Léna and Julien Léo,
To my whole family, heartily*

Preface

The first conference of the *Geometric Sciences of Information* (GSI, website at <http://www.gsi2013.org/> includes slides and photos) took place downtown Paris (France), in August 2013. The call for papers received an enthusiastic worldwide response, resulting in about a 100 accepted submissions of an average length of eight pages, and organized into the following broad topics:

- Computational Information Geometry,
- Hessian/Symplectic Information Geometry,
- Optimization on Matrix Manifolds,
- Probability on Manifolds,
- Optimal Transport Geometry,
- Divergence Geometry and Ancillarity,
- Machine/Manifold/Topology Learning,
- Tensor-Valued Mathematical Morphology,
- Differential Geometry in Signal Processing,
- Geometry of Audio Processing,
- Geometry for Inverse Problems,
- Shape Spaces: Geometry and Statistic,
- Geometry of Shape Variability,
- Relational Metric,
- Discrete Metric Spaces,
- Etc.

The GSI proceedings has been published as a (thick!) Springer LNCS (volume number 8085, XX, 879 pages, 157 illustrations). Since we could not accommodate long papers in GSI, we decided to solicit renown researchers to contribute to a full length chapter on the latest advances in information geometry and some of its applications (like computational anatomy, image morphology, statistics, and textures, to cite a few examples). Those selected applications emphasize on algorithmic aspects when programming the *Methods of Information Geometry* [1]. The 13 chapters of this book have been organized as follows:

- *Divergence Functions and Geometric Structures They Induce on a Manifold* by Jun Zhang,
- *Geometry on Positive Definite Matrices Deformed by V-Potentials and Its Submanifold Structure* by Atsumi Ohara and Shinto Eguchi,
- *Hessian Structures and Divergence Functions on Deformed Exponential Families* by Hiroshi Matsuzoe and Masayuki Henmi,
- *Harmonic Maps Relative to α -Connections* by Keiko Uohashi,
- *A Riemannian Geometry in the q -Exponential Banach Manifold Induced by q -Divergences* by Héctor R. Quiceno, Gabriel I. Loaiza and Juan C. Arango,
- *Computational Algebraic Methods in Efficient Estimation* by Kei Kobayashi and Henry P. Wynn,
- *Eidetic Reduction of Information Geometry Through Legendre Duality of Koszul Characteristic Function and Entropy: From Massieu–Duhem Potentials to Geometric Souriau Temperature and Balian Quantum Fisher Metric* by Frédéric Barbaresco,
- *Distances on Spaces of High-Dimensional Linear Stochastic Processes: A Survey*, by Bijan Afsari and René Vidal,
- *Discrete Ladders for Parallel Transport in Transformation Groups with an Affine Connection Structure* by Marco Lorenzi and Xavier Pennec,
- *A Diffeomorphic Iterative Centroid Method*, by Claire Cury, Joan A. Glaunès and Olivier Colliot,
- *Hartigan’s Method for k -MLE: Mixture Modeling with Wishart Distributions and Its Application to Motion Retrieval* by Christophe Saint-Jean and Frank Nielsen,
- *Morphological Processing of Univariate Gaussian Distribution-Valued Images Based on Poincaré Upper-Half Plane Representation* by Jesús Angulo and Santiago Velasco-Forero,
- *Dimensionality Reduction for Classification of Stochastic Texture Images* by C. T. J. Dodson and W. W. Sampson.

There is an exciting time ahead for computational information geometry in studying the fundamental concepts and relationships of Information, Geometry, and Computation!

Acknowledgments

First of all, I would like to thank the chapter contributors for providing us with the latest advances in information geometry, its computational methods, and applications. I express my gratitude to the peer reviewers for their careful feedback that led to this polished, revised work. Each chapter received from two to eight review reports, with an average number of about three to five reviews per chapter.

I thank the following reviewers (in alphabetical order of their first name): Akimichi Takemura, Andrew Wood, Anoop Cherian, Arnaud Dessein, Atsumi Ohara, Bijan Afsari, Frank Critchley, Frank Nielsen, Giovanni Pistone, Hajime

Urakawa, Hirohiko Shima, Hiroshi Matsuzoe, Hitoshi Furuhashi, Isabelle Bloch, Keiko Uohashi, Lipeng Ning, Manfred Deistler, Masatoshi Funabashi, Mauro Dalla-Mura, Olivier Alata, Richard Nock, Silvère Bonnabel, Stéphanie All-
assonnière, Stefan Sommer, Stephen Marsland, Takashi Kurose, Tryphon
T. Georgiou, Yoshihiro Ohnita, and Yu Fujimoto.

I would also like to reiterate my warmest thanks to our scientific, organizing, and financial sponsors: CNRS (GdRs MIA and Maths and Entreprises), Ecole des Mines de Paris, Supelec, Université Paris-Sud, Institut Mathématique de Bordeaux, Société de l'Electricité, de l'Électronique et des technologies de l'information et de la communication (SEE), Société Mathématique de France (SMF), Sony Computer Science Laboratories, and Thales.

Last but not least, I am personally indebted to Dr. Mario Tokoro and Dr. Hiroaki Kitano (Sony Computer Science Laboratories, Inc) for their many encouragements and continuing guidance over the years.

Tokyo, January 2014

Frank Nielsen

Reference

1. Amari, S., Nagaoka, H.: Method of information geometry, AMS Monograph. Oxford University Press, Oxford (2000)

Contents

1	Divergence Functions and Geometric Structures They Induce on a Manifold.	1
	Jun Zhang	
2	Geometry on Positive Definite Matrices Deformed by V-Potentials and Its Submanifold Structure	31
	Atsumi Ohara and Shinto Eguchi	
3	Hessian Structures and Divergence Functions on Deformed Exponential Families	57
	Hiroshi Matsuzoe and Masayuki Henmi	
4	Harmonic Maps Relative to α-Connections	81
	Keiko Uohashi	
5	A Riemannian Geometry in the q-Exponential Banach Manifold Induced by q-Divergences	97
	Héctor R. Quiceno, Gabriel I. Loaiza and Juan C. Arango	
6	Computational Algebraic Methods in Efficient Estimation	119
	Kei Kobayashi and Henry P. Wynn	
7	Eidetic Reduction of Information Geometry Through Legendre Duality of Koszul Characteristic Function and Entropy: From Massieu–Duhem Potentials to Geometric Souriau Temperature and Balian Quantum Fisher Metric	141
	Frédéric Barbaresco	
8	Distances on Spaces of High-Dimensional Linear Stochastic Processes: A Survey	219
	Bijan Afsari and René Vidal	

9 Discrete Ladders for Parallel Transport in Transformation Groups with an Affine Connection Structure 243
Marco Lorenzi and Xavier Pennec

10 Diffeomorphic Iterative Centroid Methods for Template Estimation on Large Datasets 273
Claire Cury, Joan Alexis Glaunès and Olivier Colliot

11 Hartigan’s Method for k -MLE: Mixture Modeling with Wishart Distributions and Its Application to Motion Retrieval 301
Christophe Saint-Jean and Frank Nielsen

12 Morphological Processing of Univariate Gaussian Distribution-Valued Images Based on Poincaré Upper-Half Plane Representation 331
Jesús Angulo and Santiago Velasco-Forero

13 Dimensionality Reduction for Classification of Stochastic Texture Images 367
C. T. J. Dodson and W. W. Sampson

Index 389