Measurement Uncertainties in Science and Technology

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Second Edition

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To the memory of my parents
in love and gratitude

Preface to the Second Edition

> Truth is truth
> to the end of reckoning
>
> *Shakespeare, Measure for Measure*

Physical units, though arbitrary by nature, quantify the structure of science and technology. Therefore, in a consistent system of units, measurands need to represent *true values*—"true" with respect to the system of units being considered complete in itself.

However, metrology suffers from a basic dilemma. For technical reasons, measuring processes are accompanied by measurement errors. This is why measured results are more or less blurred. Nevertheless, the endeavours of metrology should be traceable, i.e measurement results should localize the true values of the measurands via intervals of the kind: estimator ± uncertainty. Hence, metrology needs robust evaluation procedures—robust in the sense that the procedures spawn reliably rated uncertainty intervals.

Clearly, measuring errors cannot be treated from within themselves, say, detached from the basic operating principles of measuring devices. In practice, experimenters are faced with two hurdles. The first concerns the treatment of what C.F. Gauss termed "regular or constant errors", currently referred to as *unknown systematic errors*. The second concerns the handling of random errors.

The question of how to treat unknown systematic perturbations is unresolved. In this book, I argue that the one of the keys to a satisfactory treatment of measurement errors lies in the treatment of unknown systematic perturbations via worst-case methods.

Surprisingly enough, the influence of random errors on measurement uncertainties is equally unexplored. Indeed, when attempting to establish appropriate conditions of measurement, the scope of confidence intervals as introduced by W. Gosset ("Student") appears extendable to any number of variables, an observation that follows from a reinvestigation of the role of empirical moments of second order.

In this book, the treatment of unknown systematic errors and of random errors lead to an essentially new approach to the assignment of measurement uncertainties, the *Generalized Gaussian Error Calculus*. The central topic is the compliance of the formalism with the requirements of traceability.

A point of particular concern arising from a conceivably unified error calculus involves the plethora of practice-related ad hoc approaches, e.g. [48, 51], which can now, advantageously, be cast into a comprehensible formalism. Beyond everyday applications, say, legal or scientific purposes, shifts of the numerical values of the fundamental constants of physics may be anticipated. As the effects of physics are bound by fundamental constants, numerical shifts, if substantiated, might lead to new research verifying basic concepts.

The second edition of *Measurement Uncertainties in Science and Technology* [38] orders and restructures the text of the first edition from scratch. Greater emphasis is placed on the methodology: using a range of examples, I show how to design uncertainty intervals localizing the true values of the measurands (i.e."true" in view of their relationship to the adopted system of physical units). The examples demonstrate the efficiency and reliability of the procedure. As ever, suggestions and comments are very welcome.

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Preface to the First Edition

In his treatises on the Method of Least Squares C.F. Gauß [1] distinguished irregular or random errors on the one hand and regular or constant errors on the other. As is well known, Gauß excluded the latter from his considerations on the ground that the observer should either eliminate their ultimate causes or rid every single measured value of their influence.

Today, these regular or constant errors are called unknown systematic errors or, more simply, systematic errors. Such errors turned out, however, to be eliminable neither by appropriately adjusting the experimental set-ups nor by any other means. Moreover, considering the present state of measurement technique, they are of an order of magnitude comparable to that of random errors.

The papers by C. Eisenhart [9] and S. Wagner [10] in particular have entrusted the high-ranking problem of unknown systematic errors to the metrological community. But it was not until the late 1970s, that it took center stage apparently in the wake of a seminar held at the Physikalisch-Technische Bundesanstalt in Braunschweig [20]. At that time two ways of formalizing unknown systematic errors were discussed. One of these suggested including them smoothly via a probabilistic artifice into the classical Gaussian calculus and the other, conversely, proposed generalizing that formalism in order to better bring their particular features to bear.

As the author prefers to see it systematic errors introduce biases and this situation would compel the experimenter to differentiate between expectations on the one hand and true values on the other—a distinction that does not exist in the conventional error calculus. This perspective and another reason, which will be explained below, have induced the author to propose a generalization of the classical error calculus concepts. Admittedly, his considerations differ substantially from those recommended by the official metrology institutions. Today, the latter call for international validity under the heading *Guide to the Expression of Uncertainty in Measurement* [41–43].

Meanwhile, both formal and experimental considerations have raised numerous questions: The *Guide* does not make a distinction between true values and expectations; in particular, uncertainty intervals are not required to localize the true values of the measurands. Nonetheless, physical laws in general as well as interrelations between physical constants in particular are to be expressed in terms of true values. Therefore, a metrology which is not aimed at accounting for a system of true values is scarcely conceivable.

As the *Guide* treats random and systematic errors in like manner on a probabilistic basis, hypothesis testing and analysis of variance should remain valid; in least squares adjustments, the minimized sum of squared residuals should approximate the number of the degrees of freedom of the linear system. However, all these assumptions do not withstand a detailed analysis.

In contrast to this, the alternative error model to be discussed here suggests healing answers and procedures appearing apt to overcome the said difficulties.

In addition to this, the proposal of the author differs from the recommendations of the *Guide* in another respect, inasmuch as it provides the introduction of what may be called "well-defined measurement conditions". This means that each of the measurands, to be linked within a joint error propagation, should be subjected to the same number of repeated measurements. As obvious as this might seem, the author wishes to boldly point out that just this procedure would return the error calculus back to the womb of statistics which it had left upon its way through the course of time. Well-defined measurement conditions allow complete empirical variance–covariance matrices to be assigned to the input data and this, in fact, offers the possibility of expressing that part of the overall uncertainty which is due to random errors by means of Student's statistics.

Though this idea is inconsistent with the traditional notions of the experimenters which have at all times been referred to incomplete sets of data, the attainable advantages when reformulating the tools of data evaluation in terms of the classical laws of statistics appear to be convincing.

After all, there is another point worth mentioning: the approach to always assess the true values of the measurands in general and the physical constants in particular may be seen to endorse, quasi a priori, the fundamental demand of metrology for *traceability*. Actually, the gist of this definition implies nothing else but just that what has been stated above, namely the localization of true values. While the *Guide* cannot guarantee traceability, this is the basic property of the alternative error model referred to here.

Last but not least, I would like to express my appreciation to my colleagues' experience which I referred to when preparing the manuscript, as well as their criticism which inspired me to clarify the text. For technical support I am grateful to Dr. Michael Weyrauch and to Dipl.-Übers. Hannelore Mewes.

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