

Studies in Fuzziness and Soft Computing

Volume 314

Series editors

Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

For further volumes:

<http://www.springer.com/series/2941>

About this Series

The series “Studies in Fuzziness and Soft Computing” contains publications on various topics in the area of soft computing, which include fuzzy sets, rough sets, neural networks, evolutionary computation, probabilistic and evidential reasoning, multi-valued logic, and related fields. The publications within “Studies in Fuzziness and Soft Computing” are primarily monographs and edited volumes. They cover significant recent developments in the field, both of a foundational and applicable character. An important feature of the series is its short publication time and world-wide distribution. This permits a rapid and broad dissemination of research results.

Zeshui Xu

Hesitant Fuzzy Sets Theory

 Springer

Zeshui Xu
Business School
Sichuang University
Chengdu
Sichuang
China

ISSN 1434-9922

ISSN 1860-0808 (electronic)

ISBN 978-3-319-04710-2

ISBN 978-3-319-04711-9 (eBook)

DOI 10.1007/978-3-319-04711-9

Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013958274

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

When people make a decision, they are usually hesitant and irresolute for one thing or another which makes it difficult to reach a final agreement. For example, two decision makers discuss the membership degree of an element to a set, and one wants to assign 0.6 but the other 0.8. Accordingly, the difficulty of establishing a common membership degree is not because we have a margin of error, or some possibility distribution values, but because we have a set of possible values. To deal with such cases, Torra and Narukawa (2009) introduced the concept of hesitant fuzzy set (HFS). The HFS, as one of the extensions of Zadeh (1965)'s fuzzy set, allows the membership degree that an element to a set presented by several possible values, and can express the hesitant information more comprehensively than other extensions of fuzzy set. In 2011, Xu and Xia defined the concept of hesitant fuzzy element (HFE), which can be considered as the basic unit of a HFS, and is also a simple and effective tool used to express the decision makers' hesitant preferences in the process of decision making. Since then, our research group has done lots of research work on aggregation, distance, similarity and correlation measures, clustering analysis, and decision making with hesitant fuzzy information.

In this book, we give a thorough and systematic introduction to the main research results in hesitant fuzzy theory, which include the hesitant fuzzy aggregation techniques, the hesitant fuzzy preference relations, the hesitant fuzzy measures, the hesitant fuzzy clustering algorithms, and the hesitant fuzzy multi-attribute decision making methods, etc. We organize this book into four chapters that deal with four different but related issues, which are listed below:

Chapter 1 introduces a series of hesitant fuzzy aggregation operators. We first introduce the hesitant fuzzy elements (HFEs), give their comparison methods, basic operational laws, and their desirable properties. Based on these operations, we develop lots of operators for aggregating HFEs, such as the hesitant fuzzy weighted aggregation operators, the generalized hesitant fuzzy weighted aggregation operators, the hesitant fuzzy ordered weighted aggregation operators, the generalized hesitant fuzzy ordered weighted aggregation operators, the hesitant fuzzy hybrid aggregation operators, the generalized hesitant fuzzy hybrid aggregation operators, the hesitant fuzzy Bonferroni means, the hesitant fuzzy aggregation operators based on quasi-arithmetic means and the induced idea, the hesitant fuzzy aggregation operators based on t-norms and t-conorms, the hesitant multiplicative aggregation operators, discuss their relations in detail, and apply them to the enterprise's development planning of strategy initiatives, site

selection, the supplier selection in a supply chain, safety evaluation of work systems, etc.

Chapter 2 mainly investigates the distance, similarity, correlation, entropy measures and clustering algorithms for hesitant fuzzy information. We first introduce a series of distance measures for HFSs, based on which the corresponding similarity measures are given. Then we investigate the distance and correlation measures for HFEs, and discuss their properties in detail. We introduce the concepts of entropy and cross-entropy for hesitant fuzzy information, and analyze the relationships among the proposed entropy, cross-entropy, and similarity measures. We also introduce some correlation coefficient formulas and use them to calculate the degrees of correlation among HFSs aiming at clustering different objects. Moreover, we give the hesitant fuzzy agglomerative hierarchical clustering algorithm, the hierarchical hesitant fuzzy K-means clustering algorithm which takes the results of hierarchical clustering as the initial input, and also introduce a minimal spanning tree algorithm-based clustering technique to make clustering analysis of HFSs via some hesitant fuzzy distances. The applications of the algorithms in energy policy evaluation, medical diagnosis, supplier selection of manufacturing enterprise, software evaluation and classification, and tourism resources assessment, etc., are demonstrated.

Chapter 3 focuses on group decision making with hesitant preference relations. We introduce the concepts of hesitant fuzzy preference relation and multiplicative preference relation, by using them we give two group decision making approaches. Based on the multiplicative consistency and the acceptable multiplicative consistency, we establish two algorithms to improve the inconsistency level of a hesitant fuzzy preference relation, and investigate the consensus of group decision making based on hesitant fuzzy preference relations. We introduce two regression methods that transform hesitant fuzzy preference relations into fuzzy preference relations, which depend on the additive transitivity and the weak consistency respectively. Based on two principles (i.e., α -normalization and β -normalization), we develop a hesitant goal programming model to derive priorities from hesitant fuzzy preference relations and some consistency measures of hesitant fuzzy preference relations. Additionally, we introduce a hesitant fuzzy programming method to derive priorities from a hesitant multiplicative preference relation in AHP-hesitant group decision making.

Chapter 4 is devoted to the multi-attribute decision making models with hesitant fuzzy information. Based on the TOPSIS and the maximizing deviation method, we give an approach for solving the multi-attribute decision making problems, in which the evaluation information provided by the decision maker is expressed in HFEs and the information about attribute weights is incomplete. By using the concepts of hesitant fuzzy concordance and hesitant fuzzy discordance which are based on the given scores and the deviation degrees, we introduce a hesitant fuzzy ELECTRE I method and apply it to solve the multi-attribute decision making problem with hesitant fuzzy information. With the incomplete weight information, we define the satisfaction degree of an alternative, based on which several optimization models are derived to determine the weights of attributes, and then develop an interactive method based on some optimization models for the multi-attribute decision making problems under hesitant fuzzy

environments. Moreover, all the given methods are illustrated and used in some practical applications.

This book can be used as a reference for researchers and practitioners working in the fields of fuzzy mathematics, operations research, information science, management science and engineering, etc. It can also be used as a textbook for postgraduate and senior-year undergraduate students.

This work was supported by the National Natural Science Foundation of China under Grant 61273209.

Special thanks to Dr. Meimei Xia for providing lots of useful material.

December 2013

Zeshui Xu
Chengdu

Contents

1 Hesitant Fuzzy Aggregation Operators and Their Applications.....	1
1.1 Hesitant Fuzzy Elements.....	2
1.1.1 Comparison Methods	2
1.1.2 Basic Operations and Relations.....	6
1.2 Hesitant Fuzzy Aggregation Operators	30
1.3 Hesitant Fuzzy Bonferroni Means.....	66
1.4 Hesitant Fuzzy Geometric Bonferroni Means	80
1.5 Hesitant Fuzzy Aggregation Operators Based on Quasi-Arithmetic Means and Induced Idea.....	99
1.6 Generalized Hesitant Fuzzy Aggregation.....	132
1.7 Hesitant Multiplicative Aggregation	145
2 Distance, Similarity, Correlation, Entropy Measures and Clustering Algorithms for Hesitant Fuzzy Information	165
2.1 Distance and Similarity Measures for HFSs	169
2.2 Distance and Correlation Measures for HFEs	183
2.3 Hesitant Fuzzy Entropy and Cross-Entropy and Their Use in MADM.....	197
2.4 Correlation Coefficients of HFSs and Their Applications to Clustering Analysis	218
2.5 Hesitant Fuzzy Agglomerative Hierarchical Clustering Algorithms	235
2.6 Hierarchical Hesitant Fuzzy K-means Clustering Algorithm.....	249
2.7 MST Clustering Algorithm for HFSs.....	262
2.7.1 Graph and Minimal Spanning Trees.....	262
2.7.2 HFMST Clustering Algorithm	263
2.7.3 Numerical Examples	265
3 Hesitant Preference Relations	281
3.1 Hesitant Fuzzy Preference Relations in Group Decision Making	283
3.2 Hesitant Multiplicative Preference Relations.....	288

3.3	Transitivity and Multiplicative Consistency on Hesitant Fuzzy Preference Relation	293
3.3.1	Some Properties of Hesitant Fuzzy Preference Relation.....	294
3.3.2	Iterative Algorithm for Improving Consistency of Hesitant Fuzzy Preference Relation	298
3.3.3	Approach to Group Decision Making Based on Multiplicative Consensus of Hesitant Fuzzy Preference Relations.....	302
3.4	Regression Methods for Hesitant Fuzzy Preference Relations.....	316
3.4.1	Regression Method of Hesitant Fuzzy Preference Relations Based on Additive Transitivity.....	317
3.4.2	Regression Method of Hesitant Fuzzy Preference Relations Based on Weak Consistency	324
3.5	Deriving a Ranking from Hesitant Fuzzy Preference Relations under Group Decision Making	332
3.5.1	Deriving Priorities from Hesitant Fuzzy Preference Relations with α -Normalization	332
3.5.2	Deriving Priorities from Hesitant Fuzzy Preference Relations with β -Normalization	336
3.6	Deriving Priorities in AHP-Hesitant Group Decision Making.....	354
3.6.1	Description of the Prioritization Method.....	357
3.6.2	Hesitant Fuzzy Programming Method.....	359
3.6.3	Numerical Examples	366
4	Hesitant Fuzzy MADM Models.....	379
4.1	Hesitant Fuzzy MADM Based on TOPSIS with Incomplete Weight Information	381
4.2	ELECTRE I Method for Hesitant Fuzzy MADM	395
4.3	Interactive Decision Making Method under Hesitant Fuzzy Environment with Incomplete Weight Information	432
4.3.1	Satisfaction Degree Based Models for MADM with Incomplete Weight Information	432
4.3.2	Interactive Method for MADM under Hesitant Fuzzy Environment with Incomplete Weight Information	442
	References	449