

Springer Theses

Recognizing Outstanding Ph.D. Research

For further volumes:
<http://www.springer.com/series/8790>

Aims and Scope

The series “Springer Theses” brings together a selection of the very best Ph.D. theses from around the world and across the physical sciences. Nominated and endorsed by two recognized specialists, each published volume has been selected for its scientific excellence and the high impact of its contents for the pertinent field of research. For greater accessibility to non-specialists, the published versions include an extended introduction, as well as a foreword by the student’s supervisor explaining the special relevance of the work for the field. As a whole, the series will provide a valuable resource both for newcomers to the research fields described, and for other scientists seeking detailed background information on special questions. Finally, it provides an accredited documentation of the valuable contributions made by today’s younger generation of scientists.

Theses are accepted into the series by invited nomination only and must fulfill all of the following criteria

- They must be written in good English.
- The topic should fall within the confines of Chemistry, Physics, Earth Sciences, Engineering and related interdisciplinary fields such as Materials, Nanoscience, Chemical Engineering, Complex Systems and Biophysics.
- The work reported in the thesis must represent a significant scientific advance.
- If the thesis includes previously published material, permission to reproduce this must be gained from the respective copyright holder.
- They must have been examined and passed during the 12 months prior to nomination.
- Each thesis should include a foreword by the supervisor outlining the significance of its content.
- The theses should have a clearly defined structure including an introduction accessible to scientists not expert in that particular field.

Christian Friedrich Steinwachs

Non-minimal Higgs Inflation and Frame Dependence in Cosmology

Doctoral Thesis accepted by
University of Cologne, Germany

 Springer

Author

Dr. Christian Friedrich Steinwachs
School of Mathematical Sciences
University of Nottingham
Nottingham
UK

Supervisor

Prof. Dr. Claus Kiefer
Institut für Theoretische Physik
Universität zu Köln
Köln
Germany

ISSN 2190-5053

ISBN 978-3-319-01841-6

DOI 10.1007/978-3-319-01842-3

Springer Cham Heidelberg New York Dordrecht London

ISSN 2190-5061 (electronic)

ISBN 978-3-319-01842-3 (eBook)

Library of Congress Control Number: 2013947361

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Publications Based on this Work

- 1) Barvinsky, A.O., Kamenshchik, A.Yu., Kiefer, C., Starobinsky, A.A., Steinwachs, C.: Asymptotic freedom in inflationary cosmology with a nonminimally coupled Higgs field. *J. Cosmol. Astropart. Phys.* **12**, 003 (2009)
- 2) Barvinsky, A.O., Kamenshchik, A.Yu., Kiefer, C., Starobinsky, A.A., Steinwachs, C.: *The Higgs Field as an Inflaton*. In *Proceedings of the 12th Marcel Grossmann Meeting On General Relativity* (ed. T. Damour, R.T. Jantzen, R. Ruffini), pp. 1244–1246 (2009)
- 3) Barvinsky, A.O., Kamenshchik, A.Yu., Kiefer, C., Steinwachs, C.F.: Tunneling cosmological state revisited: Origin of inflation with a nonminimally coupled Standard Model Higgs inflaton. *Phys. Rev. D* **81**, 043530 (2010)
- 4) Steinwachs, C.F., Kamenshchik, A.Yu.: One-loop divergences for gravity nonminimally coupled to a multiplet of scalar fields: Calculation in the Jordan frame. I. The main results. *Phys. Rev. D* **84**, 024026 (2011)
- 5) Barvinsky, A.O., Kamenshchik, A.Yu., Kiefer, C., Starobinsky, A.A., Steinwachs, C.: Higgs boson, renormalization group, and naturalness in cosmology. *Eur. Phys. J. C* **72**, 2219 (2012)
- 6) Steinwachs, C.F., Kamenshchik, A.Yu.: One-loop divergences for gravity nonminimally coupled to a multiplet of scalar fields: Calculation in the Jordan frame. II. Comparison with the Einstein frame results. (in preparation)

Supervisor's Foreword

The field of cosmology is undergoing an exciting phase of development. Observations have reached an amount of precision that can compete with the precision known from other fields of physics. The most recent manifestation of this state is the release of data from the PLANCK mission in March 2013. An important aspect of current cosmology is its deep connection with particle physics and quantum theory.

The present book gives an impressive testimony of this connection. It is based on the author's Ph.D. thesis, which he completed at the University of Cologne in 2012. Two of the main features of current cosmology have to do with an accelerated expansion of the Universe. As in particular the observations of type Ia supernovae indicate, our Universe is presently accelerating. The reason for this may be a positive cosmological constant or a so far unknown form of dark energy. The second type of acceleration is more speculative and has probably happened when the Universe was younger than 10^{-30} s. This phase is called inflation. Although there is strong support for inflation for both conceptual and empirical reasons, the exact mechanism leading to inflation has not yet been revealed. Most models make use of one or more scalar fields, whose dynamics is more or less introduced in an ad hoc way.

In this book, the idea is entertained that the scalar field responsible for inflation is the one scalar field that has so far been observed: the Higgs field of the Standard Model of the strong and electroweak interaction. The corresponding Higgs particle was detected at the LHC in Geneva in 2012 and possesses a mass of around 126 GeV. As has been clear for some time, *Higgs inflation* will only work if an additional coupling of the Higgs field to gravity is included—a strong non-minimal coupling of the field to the Ricci scalar. The consequences of this coupling are the main topic of this monograph.

In the first half of his book, Christian Steinwachs presents a detailed and pedagogical introduction to all the concepts that are needed to understand modern cosmology in general and Higgs inflation in particular. These include, on the one side, the Friedmann models, the inflationary scenario, and cosmological perturbation theory, and, on the other side, the Standard Model of particle physics, effective actions, and renormalization group equations. This part of the book is

self-contained and can be read as an accessible introduction into the field itself, even if one is not interested in Higgs inflation.

In the second half, the mechanism of Higgs inflation is discussed at length, and its merits and shortcomings are critically evaluated. This part also addresses the more general question of the frame equivalence in cosmology, in particular that of the JORDAN and the EINSTEIN frame. It is shown that an equivalence is not present at the one-loop level of quantum field theory, and consequences are drawn from this result. I have so far not seen a clearer discussion of this point in the literature.

The monograph concludes with an important section on quantum cosmology. After a concise introduction into this field, it is shown that a certain boundary condition leads to a wave function that is peaked around a sufficiently high initial value of the inflaton field, which is necessary for inflation to occur in the first place.

The book is recognizable for its interdisciplinary character and should be of interest for cosmologists and particle physicists. I am sure it will find many readers.

University of Cologne, Cologne, June 2013

Prof. Dr. Claus Kiefer

Abstract

In this thesis, we investigate the cosmological model of non-minimal Higgs inflation. The basic idea is to unify the “low-energy” sector of the Standard Model with the high-energy phase of inflation during the early universe. Both sectors are described by different theories and rely on the existence of a fundamental scalar field. The scalar Higgs boson is an integral part of the Standard Model in order to maintain gauge invariance while explaining the origin of the masses of elementary particles. Likewise, in cosmology the scalar inflaton is necessary to describe inflation, the early phase of accelerated expansion of the universe. The basic assumption of the Higgs inflation scenario is that the two scalar particles are manifestations of one and the same particle, the *Higgs-inflaton*. Consistency with observational data of the cosmic microwave background radiation requires the assumption of a strong non-minimal coupling of the Higgs-inflaton to gravity. It turns out that quantum corrections of the heavy Standard Model particles mainly determine the cosmological parameters of the model and are crucial for the predictions. Moreover, we resort to the renormalization group flow in order to connect the energy scale of the electroweak vacuum with the inflationary energy scale, separated by many orders of magnitude. We derive a range of possible values for the mass of the Higgs-inflaton. These predictions can be tested by future experiments at the Large Hadron Collider (LHC) and by the satellite PLANCK.

We further investigate an application of the non-minimal Higgs inflation model in the context of quantum cosmology. Via the path integral approach, we calculate the cosmological probability distribution for the universe to tunnel “from nothing to existence”. A sharp peak in this distribution—selecting a specific value of the inflaton field—can be interpreted as the initial condition for inflation. The predicted value can be tested by the detection of primordial gravitational waves.

In order to technically realize these models, we calculate the divergent part of the one-loop off-shell effective action of a $O(N)$ -symmetric multiplet of scalar fields non-minimally coupled to gravity in a closed form. Due to the functional nature of the couplings, the result is very general and can be used as a reference for many different cosmological applications.

In cosmology it is convenient to transform between two specific field parametrizations, denoted *Jordan frame* and *Einstein frame*. There is an ongoing discussion whether these two frames are physically equivalent. Mathematically, both frames are equivalent at the classical level. We explicitly show that this does

not hold anymore at the quantum level. Moreover, we identify this failure of equivalence to be part of a more general problem: the parametrization dependence of the conventional off-shell effective action. This reduces the cosmological debate between Jordan frame and Einstein frame to one problem: the non-covariance of the conventional formalism.

Acknowledgments

I would like to thank all those who supported me in completing this thesis. In particular, I would like to express my gratitude to those who guided and accompanied me during the last years:

First of all, I am indebted to the Supervisor of my thesis, Prof. Dr. Claus Kiefer. I did not only benefit from his wide range of knowledge, but I also had the privilege to learn from numerous discussions with him about fundamental conceptual questions of the cosmos and quantum theory. He introduced me to his longterm collaborators and offered me the possibility to actively participate in joint projects, while also providing me with the freedom to pursue my own scientific ideas. Moreover, he encouraged me to present the results of our common work at various international conferences and generously supported my participation.

I owe gratitude to my collaborators, who kindly shared their knowledge with me. Most notably, I am deeply obliged to Prof. Dr. Alexander Yu. Kamenshchik, who always patiently answered all my questions. I strongly benefited from his extensive skills and his helpful advice. I am also grateful for the opportunity of working with Prof. Dr. Andrei O. Barvinsky, an expert on the mathematical techniques used in this thesis. Moreover, I am indebted to Prof. Dr. Alexei A. Starobinsky for illuminating conversations. Being a leading expert in cosmology, his rich ideas and helpful comments strongly influenced the direction of our common work.

I would also like to express my gratitude to Prof. Dr. Friedrich W. Hehl, who provided me with literature about computer algebra systems and gave me valuable advice.

It is a pleasure to thank Priv.-Doz. Dr. Rochus Klesse for interesting and stimulating discussions, especially on the process of decoherence.

I am thankful to Prof. Dr. Dominik J. Schwarz and Prof. Dr. Martin Reuter for kindly accepting to be the co-referees of this thesis.

I am grateful to my friend and long-term office mate Dr. Friedemann Queißer for the numerous interesting discussions during our coffee breaks. Furthermore,

my thanks go to Manuel Krämer for his very accurate orthographic proofreading of large parts of this thesis.

I am thankful to Prof. Dr. Steven Christensen for sending me the latest version of the MathTensor package and to Dr. Andreas Sindermann for taking care of its proper installation.

Finally, I owe special thanks to the “Evangelisches Studienwerk Villigst e.V.” for the financial support of my doctoral project.

Aside from the scientific, technical and financial support I have experienced during the years, I would like to thank Simin Askari and my parents Gabriele and Burkhard Steinwachs, who constantly encouraged me to follow my interests and always provided me with their help and advice.

Contents

1	Introduction	1
2	Cosmology	5
2.1	General Relativity	5
2.1.1	Symmetry Reduction	7
2.2	Friedmann–Robertson–Walker Universe	8
2.2.1	Physical Versus Comoving Distance	9
2.2.2	Conformal Time	10
2.2.3	Friedmann Equations	11
2.2.4	Epochs of the Universe	13
2.2.5	Observable Quantities	15
2.3	Inflation	16
2.3.1	Qualitative Preliminary Considerations	17
2.3.2	Scalar Field Model of Inflation	19
2.3.3	De Sitter Space	21
2.3.4	Slow-Roll Inflation	23
2.4	Cosmological Perturbations	25
2.4.1	Generation of Density Fluctuations: Basic Mechanism	26
2.4.2	Decomposition of Different Types of Fluctuations	27
2.4.3	Quantization of Fluctuations	31
2.5	Horizon Crossing and Observable Quantities	33
2.5.1	Slow-Roll Power Spectrum	36
2.5.2	Cosmological Parameters During Slow-Roll	38
	References	40
3	Standard Model	43
3.1	Gauge Theories	45
3.1.1	Electromagnetic Interaction $U_{EM}(1)$	46
3.1.2	Strong Interaction $SU_c(3)$	50
3.1.3	Electroweak Interaction $SU_L(2) \times U_{Y_w}(1)$	54
3.2	Masses of Elementary Particles	58
3.2.1	Massive Fermions in the Standard Model?	58
3.2.2	The Massless Standard Model Lagrangian	60

3.2.3	Spontaneous Symmetry Breaking	62
3.2.4	Higgs Mechanism and the Generation of Masses	66
	References	70
4	Effective Action and Renormalization Group	71
4.1	Classical Field Theory and Dimensional Analysis	71
4.2	Divergences, Regularization and Renormalization	73
4.3	Renormalization Group	76
4.3.1	Illustrative Examples	76
4.3.2	Gell–Mann–Low Equations	77
4.3.3	Relation Between Counterterms and Beta Functions	80
4.4	Path Integral and Effective Action	82
4.4.1	Faddeev–Popov–Method	86
4.5	Generalized Schwinger–DeWitt Algorithm	90
4.5.1	Reduction Algorithm for Minimal Operators	91
4.5.2	Dimensional Regularization and the \hat{a}_2 Coefficient	95
4.5.3	Recursion Relation and Coincidence Limits	96
	References	97
5	One-Loop Cosmology and Frame Dependence	99
5.1	Minimally Coupled Single Field in the Einstein Frame	100
5.1.1	Calculation of the Fluctuation Operator	100
5.1.2	Calculation of the Minimal Operator	105
5.1.3	Results	107
5.1.4	Comparison with 't Hooft and Veltman	112
5.2	Non-minimal $O(N)$ Multiplet in the Jordan Frame	113
5.2.1	Second Variation and Minimal Operator	114
5.2.2	One-Loop Divergences in the Jordan Frame: The Result	119
5.2.3	Checks and Comparison with Known Results	127
5.3	Cosmological Debate: Jordan Frame Versus Einstein Frame	132
5.3.1	Transition Between the Frames	134
5.3.2	Frame Dependence of Quantum Corrections	140
5.3.3	Geometrical Effective Action	146
5.4	Conclusions	150
	References	151
6	Non-minimal Higgs Inflation	153
6.1	Motivation for a Non-minimal Coupling	154
6.1.1	Minimal Higgs Inflation	154
6.1.2	Non-minimal Higgs Inflation	155
6.1.3	Beyond General Relativity	156
6.2	The Standard Model Higgs Boson as the Inflaton	158
6.2.1	Matter Sector	158
6.2.2	Graviton-Scalar Sector	160

6.2.3	S-Factor Suppression	161
6.2.4	Origin of Suppression and Goldstone Contributions	163
6.3	Cosmological Aspects of the Model	165
6.3.1	Slow-roll Dynamics in the Jordan Frame	165
6.3.2	Effective Potential and Slow-Roll in the Einstein Frame	167
6.3.3	Cosmological Parameters in Non-minimal Higgs Inflation	170
6.4	Higgs Inflation and Renormalization Group	171
6.4.1	Higgs Inflation at the Electroweak Scale	171
6.4.2	Renormalization Group Improvement	172
6.4.3	Diagrammatic Insertion Technique of s-Factors	175
6.4.4	Running Potential at the Inflationary Scale	179
6.5	Numerical Analysis	182
6.5.1	Exact Numerical Solutions of the RG Improved Model	183
6.5.2	Discussion of Numerical Results	186
6.6	Validity of the Model: Naturalness and Unitarity	192
6.6.1	Running Cut-off	192
6.6.2	Scale-Invariance and Shift-Symmetry	194
6.7	Variants of Higgs Inflation	198
6.8	Conclusions	199
	References	202
7	Quantum Cosmology	207
7.1	Conceptual Questions	207
7.1.1	Observations and Speculations	207
7.1.2	Decoherence and the Everett Many-Worlds Interpretation	209
7.1.3	Eternal Inflation and Multiverses	215
7.2	Boundary Conditions of the Cosmos	217
7.2.1	Tunnelling Condition in Minisuperspace	218
7.2.2	Beyond the Minisuperspace Treatment	222
7.2.3	Path Integral for the Microcanonical Statistical Sum	224
7.3	Quantum Origin of the Universe in the Higgs Inflation Scenario	229
7.4	Conclusions	233
	References	234
8	Summary and Outlook	239
	References	241

Appendix A: Riemann Curvature Tensor 243

Appendix B: Variations and Derivatives 247

Appendix C: Young Tableaux for $SU(3)$ 253

Appendix D: Synge’s World Function 257

Appendix E: Corrections to the Initial Values of λ and y_t 265

Appendix F: Transfer Equations 267

Appendix G: Gradient Structures 269