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Fadil Santosa, Director of the IMA

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Editors

Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations

2012 John H Barrett Memorial Lectures

 Springer

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Foreword

This volume was based on the 2012 Barrett Lectures on “Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations (PDEs)” which was partially funded by the Institute for Mathematics and its Applications (IMA) at the University of Minnesota. The workshop took place at the University of Tennessee at Knoxville from May 9–11, 2012. We would like to thank all the participants for making this a stimulating and productive workshop. In particular we would like to thank the organizers, Xiaobing Feng, Ohannes Karakashian and Yulong Xing for organizing this volume which came out of the invited speakers at the workshop.

We also take this opportunity to thank the National Science Foundation for its support of the IMA.

Minneapolis, MN

Fadil Santosa
Jiaping Wang

Preface

The John H. Barrett Memorial Lectures at the University of Tennessee, Knoxville (UTK) were established in honor of John H. Barrett, a distinguished researcher in ordinary differential equations and department head, at the time of his death in 1969. The Lectures have been given annually since 1970 in a variety of mathematical fields by a succession of distinguished lecturers. The topic of the Barrett Lectures changes from year to year and is chosen to reflect research interests within the Department of Mathematics at the University of Tennessee. Since 1993 the Lectures have consisted of two or three one-hour survey talks by each of two or three leading researchers, representing different themes/directions in a single field. The Lectures are partly funded by a grant from Mathematics Department of the University of Tennessee and have often received additional support from the National Science Foundation. They attract wide interest, with an audience of between 40 and 60 participants from the whole country, in addition to faculty and students from UTK and the Oak Ridge National Laboratory. They represent one of the few long-standing lecture series in mathematics in the southeastern USA.

The 2012 Barrett Lectures, which is the 42nd Lecture in the series, were held on the campus of the University of Tennessee at Knoxville from May 9–11, 2012. The topic of the 2012 Barrett Lectures was “Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations (PDEs),” which is a hot topic and has a broad appeal to researchers from applied sciences and engineering. One of the primary goals of the Barrett Lectures is to bring prominent researchers working in such active areas to UTK, as a service to the university and to the southeastern region of the USA. As one of the few long running lecture series in mathematics in the southeastern USA, plus the popularity and broad appeal of its topic, it was expected that there was a large attendance of the Lectures in 2012. About 70 people from the USA and Europe attended

the Lectures and half of the attendees are junior researchers (assistant professors, postdocs, and graduate students).

The main speakers of the 2012 Barrett Lectures were *Franco Brezzi* of University of Pavia (Italy) and *Chi-Wang Shu* of Brown University. Each of them delivered three one-hour survey lectures with Franco Brezzi focusing on theoretical aspects of discontinuous Galerkin methods for elliptic problems and Chi-Wang Shu on discontinuous Galerkin methods for evolution problems and their applications. The titles of their talks were:

1. Franco Brezzi (University of Pavia, Italy)
 - Part I: Theoretical aspects of discontinuous Galerkin (DG) methods for stationary problems: mathematical background of DG methods.
 - Part II: Classical DG methods for second and fourth order elliptic problems.
 - Part III: Connections between DG and other methods.
2. Chi-Wang Shu (Brown University)
 - Discontinuous Galerkin methods for time-dependent problems: survey and recent developments: Parts I–III

In addition to the two main speakers, there were also ten one-hour invited talks for the 2012 Lectures. These ten speakers and titles of their talks were

1. Slimane Adjerid (Virginia Tech): Accurate error estimates and superconvergence for DG methods.
2. Susanne Brenner (Louisiana State University): C^0 interior penalty methods.
3. Bernardo Cockburn (University of Minnesota): Devising superconvergent DG methods.
4. Clint Dawson (University of Texas at Austin): Local time stepping in DG methods and applications to the shallow water equations.
5. Leszek Demkowicz (University of Texas at Austin): Discontinuous Petrov–Galerkin methods with optimal test functions.
6. Jean-Luc Guermond (Texas A & M University): Discontinuous Galerkin methods for the radiative transport equation.
7. Donatella Marini (University of Pavia, Italy): Virtual elements and DG.
8. Charalambos Makridakis (University of Crete, Greece): Transport, dispersion, and local reconstructions in discontinuous Galerkin methods.
9. Ricardo Nochetto (University of Maryland): Time-discrete higher order ALE formulations: a DG approach.
10. Beatrice Riviere (Rice University): Coupled free flows and porous media flows.

This book contains articles from 11 speakers, each of whom is a leading researcher in the field of discontinuous Galerkin finite element methods and its applications. Following the tradition of the Barrett Lectures, several of these articles are in-depth survey papers with an expository discussion that should make this book a useful reference for researchers both in and out-

side the field, including other applied science and engineering communities, young researchers, and graduate students.

The 2012 Barrett Lectures were partially funded by a grant from the National Science Foundation (DMS-1203237), by the Institute for Mathematics and its Applications (IMA) at University of Minnesota, and by Research Office and College of Arts and Sciences as well as Department of Mathematics at the University of Tennessee, Knoxville. The organizers, together with all attendees, are grateful to these funding agencies. Their generous support made the Lectures possible and, among other things, allowed the organizers to fund the participation of young researchers including graduate students and recent postdocs.

Finally, we would like to express our thanks to Ms. Connie Mroz and Jane Parker, the secretaries of the 2012 Barrett Lectures, who made all organizational details run smoothly, and Juvy Melton, who helped to do the budget and all the paper work for grant applications. We would also like to thank Ben Walker, Angela Woofter, Thomas Lewis, and Cody Lorton for their various help during the Lectures.

This is the first time when the proceedings of the Barrett Lectures is published as a volume in the IMA book series. We are grateful to Professor Fadil Santosa, the director of IMA, for his enthusiasm and encouragement to publish the proceedings. We would also like to thank Katherine Cramer of IMA and Achi Dosanjh of Springer for their help during the course of the preparation.

Knoxville, TN

Xiaobing Feng
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