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Eulalia Szmidt

Distances and Similarities in Intuitionistic Fuzzy Sets

 Springer

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Foreword

The book I am glad to write my foreword to is a very relevant position in literature on intuitionistic fuzzy sets or, maybe even more generally, in the fuzzy set theory.

In virtually all application issues related to the very essence of similarity, distances are crucial. Just to quote some more important examples, let me mention data analysis and data mining, machine learning, decision theory and analysis, control etc. Of course, this short list is by no means exhaustive.

Professor Eulalia Szmidt presents in this excellent book a brief, yet an extremely informative and constructive account of various tools and techniques to effectively and efficiently define and determine similarity in the case of objects described in terms of intuitionistic fuzzy sets.

The problem of quantifying similarity and distance can be viewed from at least two perspectives. On the one hand it is a conceptual level, and Professor Szmidt provides here a survey of both well known and original new concepts and measures. On the other hand, the calculations of similarity, dissimilarity, and distance in non trivial cases have an explicit algorithmic character, and many computational problems may appear. Here, again, Professor Szmidt provides the reader with a full-fledged set of procedures that can be used to solve practical problems.

To summarize, we should congratulate the Author for writing an excellent book, which certainly constitutes a part of the literature that will be widely used by the scientific community.

Sofia, June 2013

Krassimir Atanassov

Contents

1	Introduction	1
2	Intuitionistic Fuzzy Sets as a Generalization of Fuzzy Sets	7
2.1	Main Definitions	7
2.1.1	Crisp Sets (Classical Sets)	7
2.1.2	Fuzzy Sets	8
2.1.3	Intuitionistic Fuzzy Sets	8
2.2	Brief Introduction to Fuzzy Sets	10
2.2.1	Basic Concepts	10
2.2.2	Selected Operations on Fuzzy Sets	11
2.3	Brief Introduction to the Intuitionistic Fuzzy Sets	13
2.3.1	Two Geometrical Representations of the Intuitionistic Fuzzy Sets	14
2.3.2	Operations Over the Intuitionistic Fuzzy Sets	17
2.3.3	Intuitionistic Fuzzy Relations	23
2.4	Interrelationships: Crisp Sets, Fuzzy Sets, Intuitionistic Fuzzy Sets	24
2.5	Deriving the Intuitionistic Fuzzy Sets from Data	27
2.5.1	Derivation of the Intuitionistic Fuzzy Sets by Experts	30
2.5.2	Automatic Method of Deriving Intuitionistic Fuzzy Sets from Relative Frequency Distributions (Histograms)	32
2.6	Concluding Remarks	38
3	Distances	39
3.1	Basic Definitions	39
3.2	Norms and Metrics Over the Intuitionistic Fuzzy Sets or their Elements – The Two Term Approach	43
3.3	Distances between the Intuitionistic Fuzzy Sets – The Three Term Approach	52

3.3.1	Distances between the Fuzzy Sets	53
3.3.2	Distances between the Intuitionistic Fuzzy Sets	59
3.4	Ranking of the Intuitionistic Fuzzy Alternatives	73
3.5	Concluding Remarks	85
4	Similarity Measures between Intuitionistic Fuzzy Sets	87
4.1	Similarity Measures and Their Axiomatic Relation to Distance Measures	87
4.2	Some Other Counter-Intuitive Results Given by the Traditional Similarity Measures	96
4.2.1	Why the Measures Presented May Yield Counter-Intuitive Results?	100
4.3	An Example of Intuitively Justified and Operational Similarity Measure	101
4.3.1	Analysis of Agreement in a Group of Experts	103
4.4	More Examples of Similarity Measures Including the Notion of Complement	107
4.5	Correlation of Intuitionistic Fuzzy Sets	116
4.6	Concluding Remarks	128
5	Summary and Conclusions	131
	References	133
	Index	147