Lecture Notes in Computational Science and Engineering

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Uncertainty Quantification in Computational Fluid Dynamics

Springer

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Preface

The present volume addresses methods and computational aspects of efficient Uncertainty Quantification (UQ) in Computational Fluid Dynamics (CFD). While the general area of computational uncertainty quantification in engineering simulations has experienced a massive development in recent years, and is under strong expansion currently, by now key computational issues have been identified and analysis and implementation have progressed to the point where, for several broad classes of PDEs with uncertainty, computational methodologies are available which are also backed by numerical and mathematical analysis. Against this background, and consistent with the scientific focus of the 2011 Von Karmann Institute workshop which initiated the development of the chapters in the present volume, the present volume combines several contributions on efficient methods for UQ in CFD which address specific computational issues which arise in the use of the general computational methodologies in CFD problems; some (but not all) of these are: highly nonlinear, unsteady nature of the governing equations, singularity (shock) formation in finite time in pathwise solutions, and the impact of discontinuities on the accuracy and the regularity of statistical quantities even when all data in the problem are smooth. Other issues are the corresponding low regularity in the space of uncertain parameters, massive parallelism in forward simulations, necessity for multiscaling and multi-modelling in forward simulations (in particular in the presence of turbulence), uncertain topography and geometry of the flow domain.

The low solution regularity and the propagation of singularities in solutions of the governing equations prompt the development of the numerical techniques which are specifically adapted to deal with these pheonomena; among them are Finite Volume methods in the stochastic parameter domain, WENO reconstruction and limiting methods for positivity enforcement in computation of probability density functions of random solution quantities, to name but a few. Most of these techniques are *nonintrusive* since, unlike the situation encountered in computational UQ in solids and wave propagation, the strong nonlinearity of the governing equation narrows applicable UQ methods to essentially those of collocation type. Due to the low, pathwise regularity of solutions of nonlinear hyperbolic conservation laws, however, the (in general high) regularity properties of parametric stochastic solutions required for example by spectral collocation methods must be carefully verified in practice. A logical conclusion of these remarks is the prominent role which will be played by *stochastic collocation* methods and by *Monte-Carlo* sampling approaches; in particular, *Multilevel Monte-Carlo* sampling approaches have proved quite efficient and powerful strategies when solving UQ problems in CFD. We are confident that the methods which we found to be viable and robust for the CFD problems considered here will also prove to be applicable to other, "hard" and fully nonlinear computational models in engineering and in the sciences.

The notes address computational technicalities of specific issues arising in UQ in current CFD applications, in particular UQ in output functionals such as lift-, drag- and other, integral quantities of the primitive uncertain variables, estimation of statistical moments, in particular of variance, and the probability of computation of extremal events, and the assessment of the accuracy of statistical quantities in the presence of discretization and other, numerical errors.

While these notes focus on computational and implementation aspects of discretization, stability, parallelization of computational UQ for problems in CFD. Naturally, they impinge on a number of related issues in numerical analysis and high performance computing; we only mention load balancing issues in massively parallel UQ simulations and the mathematical regularity of statistical quantities of output functionals; here, the most prominent example is that of statistics of shock locations and profiles where additional regularity of outputs is generated by ensemble averaging of random entropy solutions, so that for example the statistics of shock locations can become Lipschitz continuous or more regular, even for hyperbolic equations without any viscosity.

As can be expected in a field which is currently undergoing rapid development, the present notes represent only a snapshot in time of this evolving field of computational science and engineering. The present notes are intended to present the key ideas, the description of UQ algorithms, and prototypical implementations on a high technical level, which should be accessible, nevertheless by graduate students and researchers in computational science as well as in CFD-related areas of engineering.

The background knowledge of the intended readership of this volume is knowledge of elementary probability and statistics and solid knowledge of computational fluid dynamics.

We very much hope that these notes stimulate further algorithmic and theoretical developments in UQ for CFD and, due to the interdisciplinarity nature of UQ, also in the adjacent areas of statistics, high-performance computing, and the analysis of partial differential equations with random input data.

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