

# Dynamics of Vortex Structures in a Stratified Rotating Fluid

# ATMOSPHERIC AND OCEANOGRAPHIC SCIENCES LIBRARY

---

VOLUME 47

---

## *Editor*

Lawrence A. Mysak, *Department of Atmospheric and Oceanographic Sciences, McGill University, Montreal, Canada*

## *Editorial Advisory Board*

A. Berger	Université Catholique, Louvain, Belgium
J.R. Garratt	CSIRO, Aspendale, Victoria, Australia
J. Hansen	MIT, Cambridge, MA, U.S.A.
M. Hantel	Universität Wien, Austria
H. Kelder	KNMI (Royal Netherlands Meteorological Institute), De Bilt, The Netherlands
T.N. Krishnamurti	The Florida State University, Tallahassee, FL, U.S.A.
P. Lemke	Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany
A. Robock	Rutgers University, New Brunswick, NJ, U.S.A.
S.H. Schneider <sup>†</sup>	Stanford University, CA, U.S.A.
G.E. Swaters	University of Alberta, Edmonton, Canada
J.C. Wyngaard	Pennsylvania State University, University Park, PA, U.S.A.

For further volumes:

<http://www.springer.com/series/5669>

Mikhail A. Sokolovskiy • Jacques Verron

# Dynamics of Vortex Structures in a Stratified Rotating Fluid

 Springer

Mikhail A. Sokolovskiy  
RAS, Water Problems Institute  
Moscow, Russia

Jacques Verron  
CNRS  
Grenoble, France

This book is an extended edition of the translation of the book in Russian “Dinamika vikhreykh struktur v stratifitsirovannoy vraschayuscheysya zhidkosti” by M.A. Sokolovskiy and J. Verron. The book was published originally in Russian by Publishing House of Izhevsk Institute of Computer Science, Moscow in 2011.

Translated from Russian by Gennady N. Krichivets and Olga I. Yakovenko.

ISSN 1383-8601

ISBN 978-3-319-00788-5

ISBN 978-3-319-00789-2 (eBook)

DOI 10.1007/978-3-319-00789-2

Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013945855

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

*Dedicated*

*to the blessed memory of  
Professor Vadim Fedorovich Kozlov,  
the Founder of Far Eastern School  
of Geophysical Fluid Dynamics,  
Vladivostok, Russia  
(1933–2005)*

*and*

*to Emil Hopfinger,  
Directeur de Recherche CNRS,  
Ex-Director of Laboratoire des Ecoulements  
Géophysiques et Industriels,  
Grenoble, France*



# Foreword

This monograph is an expanded English translation of the Russian book published in 2011 by the Izhevsk Institute of Computer Science Publishing House under the same name and reviewed by Dr. G.M. Reznik.

The objective of this book is studying, in the framework of quasi-geostrophy, the role of stratification in the synoptic-scale vortex dynamics problems for atmosphere and ocean, and the effect of a bottom topography on large/meso-scale currents and vortices. The book summarizes the long-term joint studies of the authors in vortex dynamics of stratified rotating fluid.

The book is intended for experts in physical oceanography and meteorology, hydrodynamics, dynamical systems, and teachers, post-graduate students, and students in those fields.

Mikhail Sokolovskiy  
Water Problems Institute  
Russian Academy of Science  
3, Gubkina Str., 119333  
Moscow, Russia  
sokol@aqu.laser.ru

Jacques Verron  
Laboratoire des Ecoulements  
Géophysiques et Industriels  
UMR 5519, CNRS, BP53 X  
38041, Grenoble Cedex, France  
jacques.verron@legi.grenoble-inp.fr

Moscow-Grenoble  
March 2013





# Preface

This book is a summary of our 18-year period of joint work in the field of vortex dynamics in a stratified rotating fluid under INTAS 94-3614 (1995–1997), INTAS/AIRBUS 04-80-7297 (2009–2011), PICS 5805 (2011–2013) as well as within the framework of Groupement de Recherche Europeen (GDRE) “Regular and chaotic hydrodynamics” (2006–2009) and during mutual scientific visits in Grenoble and Moscow (1996–2012).

This monograph is an expanded English translation of the Russian book [858] published in 2011 by the Izhevsk Institute of Computer Science Publishing House under the same name and reviewed by Dr. G. M. Reznik. The translation was made by G. N. Krichivets and O. I. Yakovenko.

We have made the following changes as compared with the Russian edition of the book: Sects. 3.2.2 and 3.5.2, Chap. 4, Appendices A and B, and Index are added; Introduction, Sects. 2.2.2.1 and 3.5.3, and the list of references are expanded, most pictures are now in color, and found misprints are corrected. Appendix A is written by Emil Hopfinger. It covers the description of laboratory experiments on heton interaction which partially fit our theoretical results. In Appendix B, Mikhail Sokolovskiy tells a short biography of his teacher, Professor Vadim F. Kozlov.

Part of the results has been obtained in collaboration with V. F. Kozlov<sup>††</sup>, V. M. Gryanik, K. V. Koshel, V. G. Makarov, Z. Kizner, V. N. Zyryanov, X. Carton, P. A. Davies, E. Hopfinger, S. Valcke, B. N. Filyushkin, I. M. Vagina, and N. G. Kozhelupova, whose contributions to the work and kind permission to include their results in the book are much appreciated.

We are also grateful to A. V. Aksenov, M. S. Apfel’baum, H. Aref<sup>††</sup>, E. N. Benilov, V. L. Berdichevsky, A. V. Borisov, H. Borth, V. I. Byshev, Yu. D. Chashechkin, V. A. Cherkashin<sup>††</sup>, S. M. Corréard, V. B. Darnitskiy, E. N. Dolgopolova, F. V. Dolzhanski<sup>††</sup>, T. N. Doronina, E. V. Ermanyuk, A. I. Fel’zenbaum<sup>††</sup>, J.-B. Flór, Y. Fukumoto, R. F. Ganiev, A. I. Ginzburg, L. V. Gogish, A. N. Golubyatnikov, A. V. Gotovtsev, S. K. Gulev,

---

<sup>††</sup>Deceased.

A. Yu. Gurulev, M. A. Guzev, L. Kh. Ingel', V. M. Kaistrenko, M. V. Kalashnik, M. G. Khublaryan<sup>††</sup>, R. Khvoles, T. R. Kil'matov, V. I. Klyatskin, G. K. Korotaev, M. N. Koshlyakov, A. G. Kostyanoy, V. V. Kozlov, E. A. Kulikov, L. G. Kurakin, N. P. Kuzmina, S. S. Lappo<sup>††</sup>, L. Ya. Lyubavin, I. S. Mamaev, V. V. Meleshko<sup>††</sup>, A. P. Mirabel, T. Miyazaki, S. V. Muzylev, P. K. Newton, R. I. Nigmatulin, V. V. Novotryasov, V. L. Okulov, N. Paldor, G. N. Panin, E. G. Pavia, E. N. Pelinovsky, M. S. Permyakov, X. Perrot, S. V. Prants, V. V. Pukhnachov, A. B. Rabinovich, J. N. Reinaud, G. M. Reznik, K. A. Rogachev, P. B. Rutkevich, E. A. Ryzhov, E. A. Sagomonyan, G. I. Shapiro, A. I. Shavlyugin, G. V. Shevchenko, J. Sommeria, D. V. Stepanov, G. G. Sutyryn, T. G. Talipova, M. V. Tevs, D. V. Treschev, O. O. Trusenkova, A. N. Vul'fson, O. I. Yakovenko, V. I. Yudovich<sup>††</sup>, A. G. Zatsepin, P. O. Zavialov, V. V. Zhmur and D. V. Zyryanov for useful discussions of some problems considered in this book.

We express our gratitude to V. M. Gryanik, Z. Kizner, K. V. Koshel and O. I. Yakovenko for helpful comments on the Russian version of this book.

We are extremely grateful to Emil Hopfinger who kindly agreed to write Appendix A.

The authors hope that the presented results will be of interest to experts in geophysical fluid dynamics and physical oceanography. We will appreciate critical analysis of the results of our work.

Moscow-Grenoble  
March 2013

Mikhail Sokolovskiy  
Jacques Verron

# Contents

<b>1</b>	<b>The Introductory Chapter</b> .....	1
1.1	Introduction .....	1
1.2	The Mathematical Introduction.....	9
1.2.1	The Derivation of Potential Vortex Conservation Equations .....	9
1.2.2	Formal Solution. Integral Invariants .....	12
1.2.3	Contour Dynamics Method.....	15
1.2.4	Stationary Axisymmetric Solution .....	17
1.2.5	An Approach to Studying the Stability of a Axisymmetric Two-Layer Vortex .....	19
1.2.6	The Structure of Simplest Types of External Field.....	23
1.2.7	A Limiting Case of Discrete Vortices.....	24
1.2.8	Phase Portraits. Choreographies .....	26
1.2.9	Three-Layer Model Equations .....	29
<b>2</b>	<b>Dynamics of Discrete Vortices</b> .....	37
2.1	Two Vortices in a Two-Layer Fluid .....	38
2.2	$2A$ Vortices in a Two-Layer Fluid .....	41
2.2.1	The Case of Arbitrary $A$ .....	41
2.2.2	Case $A = 2$ .....	50
2.3	$A + 1$ Vortices in a Two-Layer Fluid .....	79
2.3.1	Vortex Structures with Zero Total Momentum at $A \geq 2$ (Free Motion) .....	79
2.3.2	Vortex Structures with Zero Total Momentum at $A \geq 2$ (Motion in an External Field) .....	85
2.3.3	The Case of Nonzero Total Momentum at $A = 2$ .....	110
2.4	Heton Structures in a Three-Layer Fluid.....	175
<b>3</b>	<b>Dynamics of Finite-Core Vortices</b> .....	179
3.1	Studying the Linear Stability of a Two-Layer Vortex .....	180
3.1.1	A Vortex with a Vertical Axis: Two Circular Vortex Patches .....	180

- 3.1.2 Annular Two-Layer Vortex: Four Circular Vortex Patches ..... 192
- 3.2 The Impact of Finite Perturbations ..... 199
  - 3.2.1 Heton with a Tilted Axis: Two Initially Circular Patches ..... 200
  - 3.2.2 Stationary Translation Hetonic V-States ..... 205
  - 3.2.3 Heton with a Vertical Axis: Two Initially Elliptic Vortex Patches ..... 220
- 3.3 Interaction Between Two Hetons ..... 228
  - 3.3.1 Two Hetons with Vertical Axes ..... 229
  - 3.3.2 Heton with a Vertical Axis and Heton with a Tilted Axis .... 234
  - 3.3.3 Two Hetons with Tilted Axes, the Case of Zero Total Momentum..... 243
  - 3.3.4 Two Hetons with Tilted Axes, the Case of Nonzero Total Momentum ..... 251
  - 3.3.5 Interaction Between a Warm and a Cold Hetons ..... 255
- 3.4 The Effect of External Field on Heton Motion ..... 270
- 3.5 Vortex Patch Dynamics in a Three-Layer Model ..... 277
  - 3.5.1 Stability Study of a Three-Layer Vortex ..... 280
  - 3.5.2 Modeling the Motion of Meddies ..... 285
  - 3.5.3 Examples of Interaction Between Three-Layer Vortices..... 313
- 4 The Concluding Chapter ..... 317**
  - 4.1 Concluding Remarks ..... 317
  - 4.2 Outlook to Heton Problems..... 320
  - 4.3 Discussion ..... 321
    - 4.3.1 On the Role of Baroclinic Vortices in the Formation of Thermohaline Structure of the Ocean ..... 322
    - 4.3.2 Bottom Topography and Vortices ..... 323
    - 4.3.3 More on Lenses ..... 323
    - 4.3.4 On Modons..... 324
- A E.J. Hopfinger. Experimental Study of Hetons ..... 325**
- B M.A. Sokolovskiy. In Memory of My Teacher ..... 329**
- References..... 337**
- Index ..... 379**