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Adhemar Bultheel

Laurent Series and their Padé Approximations

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CONTENTS

PREFACE

The Padé approximation problem is, roughly speaking, the local approximation of analytic or meromorphic functions by rational ones. It is known to be important to solve a large scale of problems in numerical analysis, linear system theory, stochastics and other fields.

There exists a vast literature on the classical Padé problem. However, these papers mostly treat the problem for functions analytic at 0 or, in a purely algebraic sense, they treat the approximation of formal power series. For certain problems however, the Padé approximation problem for formal Laurent series, rather than for formal power series seems to be a more natural basis.

In this monograph, the problem of Laurent-Padé approximation is central. In this problem a ratio of two Laurent polynomials in sought which approximates the two directions of the Laurent series simultaneously.
As a side result the two-point Padé approximation problem can be solved. In that case, two series are approximated, one is a power series in z and the other is a power series in z^{-1}. So we can approximate two, not necessarily different functions one at zero and the other at infinity.
To connect this problem to the previous one, we just have to glue the two series together to get a formal Laurent series. When the classical definition of Padé approximation is applied to a Laurent series instead of a power series, we get a numerator which is a polynomial plus infinitely many negative powers of z which are generated by the terms with negative powers in the Laurent series. Thus if the Laurent series has no negative powes of z, i.e. when it becomes a power series, the classical Padé approximant appears as a special case.

The first part of this volume (up to chapter 11) is purely algebraic. Three types of recursive algorithms are presented to find the solutions of the three types of Padé problems mentioned above. The first one is in principle the Trench-Zohar algorithm for Toeplitz matrix inversion. The second one is a nonsymmetric version of the Schur algorithm for deciding whether a function is a Schur function. Both these algorithms are based on a "Toeplitz approach", which means that Toeplitz matrices are a basic working tool, whereas the algorithms commonly used in the classical Padé literature (Euclid, Routh, Kronecker a.o) are more "Hankel minded". A third type of algorithm is of rhombus type and this is essentially the Rutishauser qd algorithm from numerical analysis.

From the engineering literature, the flow graph representation of electrical networks is used to represent the algorithms. This will be most useful in getting a visualization of the computational flow.

The recurrences of the algorithms can often be interpreted as recurrences associated with continued fractions and formal orthogonal polynomials. These interpretations are also treated to some extend, including reproducing kernels and Christoffel-Darboux-type formulas.

When the algorithms are used in a linear algebra context, formulas will be found for Toeplitz and Hankel matrix inversion and triangular factorization.

Finally, the block structure of the classical Padé table will be extended to describe the blocks of Laurent-Padé and two-point Padé tables.

The second part of the volume is analytic in nature. The Padé approximation problem is considered for meromorphic in $\mathbb{C} \setminus \{0\}$ functions. Classical and more recent results on the asymptotic behaviour of Hankel and Toeplitz determinants and the projection method are used for proving convergence in columns and rows of Padé tables. In classical Padé theory the convergence of columns is well established and the convergence of the rows can be obtained from a simple duality principle. For the Laurent-Padé problem, convergence of the columns is essentially the same as in the classical case. For the row convergence however, some new methods had to be used, and it is one of the most important results of this monograph. From these convergence results it is derived how the poles and zeros of a meromorphic function given by a Laurent series can be computed. The computation of poles is related to column convergence and the computation of zeros is related to row convergence.

Chapter 17 contains some interpretation of the Padé approximation problem in other theories like Carathéodory and Schur function classes, Szegő polynomials orthogonal on the unit circle, prediction theory and inverse scattering. My interest in the last topics was the inital motivation for writing this monograph. Therefore it is my sincere hope that this text will not only be appreciated by specialists in Padé approximation, continued fractions, orthogonal polynomials and Toeplitz matrices but also by people with a more applied mathematical background e.g. from stochastic processes, time series analysis, signal processing, linear systems theory or inverse scattering.

I realize that it will take some time for the reader who is not familiar with the subject to get accustomed to the large number of different types of polynomials, series and parameters. To help him, a quick reference of the most important formulas is included on a separate card which can be taken along as a bookmarker.

It remains to express my appreciation to the Computer Science department of the K.U.Leuven where I had the opportunity to prepare this text. The typesetting is done with the equipment and software provided by the department. Paul Levrie read an earlier draft of the manuscript. He pointed out a number of errors and made valuable suggestions for improvement of the text. Lieve Swinnen prepared a first draft for the typesetter.

I also wish to thank Prof. I. Gohberg, editor of this series who encouraged me to rewrite an internal report, which was the fetal version of this monograph, in a readable form.

The staff of Birkhäuser Verlag, Basel is thanked for expert advise in editing and production.

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