Part II: Weak convergence in Nonparametric statistics

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INTRODUCTION

Chapter 1 contains the basic ideas and results developed in recent years to understand the asymptotics of symmetric statistics, that are U–statistics, differentiable statistical functionals and multiple stochastic integrals. This theory is rather complete in the independent case. There are some open questions connected with the probabilistic behavior of these processes, especially in weakly dependent and long range dependent situations, which are of great interest in theoretical physics.

Chapter 2 on ranking methods does not provide a complete survey. The statistics treated in this chapter are simple linear rank statistics, signed rank statistics, functions of the order statistic, L– and R–estimators, rank tests for independence, rank tests under U–statistic structure, adaptive estimators and linear models. A few new results and some open problems are also included which may be solved in the near future.

The approach used in Chapter 1 is that of defining multiple stochastic integrals. Their definition goes back to an idea of N. Wiener and has been made precise by K. Ito. For differentiable statistical functions the approach is due to A.A. Filippova. We shall present the functional form of this idea, carried out in [DGK]. In Chapter 2 rank statistics are defined as linear operators acting on suitable spaces of score functions. It will be shown that these operators are continuous, thus reducing the question of asymptotic normality to extremely simple cases. In Chapter 1 we shall need advanced probability theory in form of invariance principles, especially the weak convergence of empirical distribution functions to the Kiefer process. When discussing other methods of obtaining weak convergence results using empirical processes, quantile processes or rank processes, we also rely on such type of theorems. All other facts needed (in both chapters) are elementary. The notes contain complete proofs where it is necessary, especially the main theorems are proven. It may be, however, that the reader needs to fill in some elementary computations. For this we refer to [D1] for additional information.