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# **Asymptotic Statistics**

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## Contents

Preface .....	7
<b>Part I: Asymptotic Expansions in Statistics <sup>1</sup></b>	
<b>by Rabi Bhattacharya</b>	
§1. Cramér–Egworth Expansions .....	11
§2. Expansions of Distributions of Statistics Admitting Stochastic Expansions .....	22
§3. Cornish–Fisher Expansions .....	40
§4. Second Order Efficiency .....	47
§5. Bootstrap and Edgeworth Expansion .....	54
§6. Notes and References .....	60
References .....	62
<b>Part II: Weak Convergence in Nonparametric Statistics</b>	
<b>by Manfred Denker</b>	
Introduction .....	68
Chapter 1: Symmetric Statistics	
1. Definition of Symmetric Statistics .....	69
2. A Distributional Limit Theorem .....	71
3. Two Applications .....	75
4. Probability Theory for Symmetric Statistics .....	84
References .....	87
Chapter 2: Statistics Based on Ranking Methods	
1. Definition of Rank Statistics .....	89
2. First Examples .....	91
3. The Asymptotic Distribution of Rank Statistics .....	93
4. Applications of Theorem 2.2 .....	99
5. Some Open Problems .....	112
6. Remarks on Efficiency .....	117
References .....	121

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## PREFACE

These notes are based on lectures presented during the seminar on "Asymptotic Statistics" held at Schloß Reisenburg, Günzburg, May 29–June 5, 1988. They consist of two parts, the theory of asymptotic expansions in statistics and probabilistic aspects of the asymptotic distribution theory in nonparametric statistics.

Our intention is to provide a comprehensive presentation of these two subjects, leading from elementary facts to the advanced theory and recent results. Prospects for further research are also included.

We would like to thank all participants for their stimulating discussions and their interest in the subjects, which made lecturing very pleasant. Special thanks are due to H. Zimmer for her excellent typing. We would also like to take this opportunity to express our thanks to the Gesellschaft für mathematische Forschung and to the Deutsche Mathematiker Vereinigung, especially to Professor G. Fischer, for the opportunity to present these lectures and to the Birkhäuser Verlag for the publication of these lecture notes.

R. Bhattacharya, M. Denker