ISNM
International Series of
Numerical Mathematics
Vol. 120

Integral Equations

Theory and Numerical Treatment

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Birkhäuser Verlag
Basel · Boston · Berlin

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Originally published in German under the title "Integralgleichungen. Theorie und Numerik"

This English translation has been authorized by and published in consent with B.G. Teubner Verlag, Stuttgart.

Translation and revisions made by the author.

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Deutsche Bibliothek Cataloging-in-Publication Data
Hackbusch, Wolfgang
Integral equations : theory and numerical treatment / Wolfgang Hackbusch. – Basel ; Boston ; Berlin : Birkhäuser, 1995
(International series of numerical mathematics ; Vol. 120)
Dt. Ausg. u.d.T.: Hackbusch, Wolfgang: Integralgleichungen

ISBN-13: 978-3-0348-9947-5 e-ISBN-13: 978-3-0348-9215-5
DOI: 10.1007/978-3-0348-9215-5

NE: GT

Softcover reprint of the hardcover 1st edition 1995
Camera-ready copy prepared by the author
Printed on acid-free paper produced from chlorine-free pulp
Cover design: Heinz Hiltbrunner, Basel

9 8 7 6 5 4 3 2 1

Preface

The theory of integral equations has been an active research field for many years and is based on analysis, function theory, and functional analysis. On the other hand, integral equations are of practical interest because of the «boundary integral equation method», which transforms partial differential equations on a domain into integral equations over its boundary.

This book grew out of a series of lectures given by the author at the Ruhr-Universität Bochum and the Christian-Albrecht-Universität zu Kiel to students of mathematics. The contents of the first six chapters correspond to an intensive lecture course of four hours per week for a semester. Readers of the book require background from analysis and the foundations of numerical mathematics. Knowledge of functional analysis is helpful, but to begin with some basic facts about Banach and Hilbert spaces are sufficient.

The theoretical part of this book is reduced to a minimum; in Chapters 2, 4, and 5 more importance is attached to the numerical treatment of the integral equations than to their theory. Important parts of functional analysis (e.g., the Riesz-Schauder theory) are presented without proof. We expect the reader either to be already familiar with functional analysis or to become motivated by the practical examples given here to read a book about this topic. We recall that also from a historical point of view, functional analysis was initially stimulated by the investigation of integral equations. Concerning function spaces, we restrict ourselves mostly to the classical ones of continuous or Hölder continuous functions. Sobolev spaces are almost avoided. As a consequence, the integral operators cannot be discussed as pseudo-differential operators in the required generality.

The theory of integral equations is interesting not only in itself, but its results are essential for the analysis of numerical methods. Besides existence and uniqueness statements, the theory concerns, in particular, questions of regularity and stability.

After an introduction which gathers basic facts from analysis, functional analysis, and numerical mathematics, we consider Volterra integral equations first (§2), since these are closely related to ordinary differential equations, which have typically been met in other courses. After these preliminaries, Fredholm integral equations of the second kind are investigated theoretically (§3) and then numerically in the subsequent chapters.

The numerical treatment of Fredholm integral equations of the second kind is discussed both in a chapter on discretisation methods (§4) and in one about the solution of the systems of linear equations which subsequently arise. Concerning the latter topic, §5 discusses in detail the multi-grid method after a brief comment about the method of conjugate gradients.

The fourth and most extensive chapter starts with general statements concerning convergence, consistency, and stability of discretisations (§4.1). The particular discretisation presented first is the kernel approximation (§4.2). Then we study the projection method in general (§4.3) and its most prominent examples: the collocation method (§4.4) and the Galerkin method (§4.5). Further remarks are added in §4.6. Another method outside of the projection method framework is the Nyström discretisation, which is discussed in §4.7. Finally, in §4.8, we give supplementary comments concerning different topics like, e.g., defect correction, extrapolation, and eigenvalue problems.

To introduce weakly and strongly singular integral equations, two examples are presented: Abel's integral equation (§6) and Cauchy integral equations (§7). The boundary integral equation method mentioned above is the subject of Chapter 8. Here, the analytical properties of the equations are discussed first. The numerical treatment leads us to the «boundary element method» (§9).

While the first chapters of the book may be thought of mainly as an introduction to the whole subject of integral equations, some parts of §§4-5 and, in particular, Chapter 9 give concrete hints which might be helpful for practical applications.

The last chapter (§9) is a revision of its counterpart in the original German version of this book. We have added recent results concerning numerical integration and the panel clustering method.

The exercises cited, which may also be understood as remarks without proof, are an integral part of the presentation. Should this book be used as the basis of an academic course, these exercises can be assigned as problems to students. However, the non-student reader should also try to test his comprehension by working on these exercises.

Completeness is not the aim of the bibliography. Furthermore, we tried to restrict ourselves to easily available literature.

The manuscript is written by the text system «Signum». My daughter Jana assisted in the construction of the figures. I am indebted to many students and readers of the first German edition for various helpful hints and corrections. In particular, I would like to thank my collegues I. Graham and A. Spence at the Bath University for polishing the English. I also wish to express my gratitude to Birkhäuser for their friendly cooperation.

Kiel, December 1994 W. Hackbusch

Contents

Notations

Numbers of formulae: Equations in a subchapter x.y are numerated by (x.y.1), (x.y.2), etc. The equation **(3.2.1)** is quoted by **(1)** in the same Section 3.2, while we write **(2.1)** in the other sections of chapter 3.

Numeration of Theorems etc.: All theorems, definitions, and lemmata etc. are enumerated together. The reference to a theorem etc. is analogous to what is said above. **Lemma 3.2.7** is cited as «**Lemma 7**» in Section 3.2, while in the other section of the chapter 3 it is denoted by «**Lemma 2.7**». However, §1 indicates Chapter 1 and never the sections §3.1 or §3.2.1.

Generic Constants: Often a notation like $\|K_n\| \leqslant C$ is used without a definition of C. The meaning is the following: There is a constant C such that $\|K_n\| \leqslant C$. Here, C is independent of the involved parameters (here; n). The value of C may be different at different places. If a fixed value is meant, this is indicated by an index (e.g., $\|K_n\| \leqslant C_K$).

Special Symbols, Abbreviations, and Conventions

a,b	bounds of the interval $I=[a,b]$
$\mathbb{C}$	complex numbers
C	constant (see above)
$C, C^k, C^\varkappa, C_L{}^k, \hat{C}^\varkappa$	spaces of continuous functions etc. (cf. §1.2)
$C_L{}^k$	space of Lipschitz continuous functions (cf. §1.2.2)
$\hat{C}^\varkappa$	space of Hölder or Lipschitz continuous functions (cf. §1.3.2)
cond	condition number (of a matrix), cf. (1.4.22)
D	domain of definition, domain of integration
det	determinant
dist($\boldsymbol{x}$,M)	distance of $\boldsymbol{x}\in\mathbb{R}^d$ from $M\subset\mathbb{R}^d$
f	unknown function of the integral equation (cf. (1.1.3))
f_n	semidiscrete solution of the integral equation
g	known function (inhomogeneity) in the integral equation (cf. (1.1.3))
h	step size of the discretisation
H^k	Sobolev space
I	interval (of integration); identity (identical operator)
$\boldsymbol{I}$	identity matrix
image(T)	range (image space) of the operator T

k	kernel function in K (cf. (3.1.3))
K	integral operator
$K(X,Y)$	compact linear mapping from X into Y
K_n	semidiscrete integral operator (cf. §4.1.1)
$\mathbf{K}_n$	matrix of the discretisation (cf. §5.1.1)
$K_r(x)$	ball with radius r centred at x
ℓ	level of discretisation (cf. §5.3)
L^2, L^1, L^∞	function spaces (cf. §1.3.3)
$L_i, L_{i,n}$	Lagrange functions
$L(X,Y)$	bounded linear mapping from X into Y
log	natural logarithm
n	discretisation parameter
$\mathbf{n}, \mathbf{n}(\mathbf{x})$	normal direction at the point $\mathbf{x}$ of the curve or surface
$\mathbb{N}$	natural numbers $\{1,2,3,...\}$
$\mathbb{N}_0$	$\mathbb{N}\cup\{0\}=\{0,1,2,...\}$
$O(\cdot)$	Landau symbol: $f(\alpha)=O(g(\alpha))$, if $\lvert f(\alpha)\rvert\leq C\lvert g(\alpha)\rvert$ with respect to an underlying limit process $\alpha\to 0$ or $\alpha\to\infty$
$o(\cdot)$	Landau symbol: $f(\alpha)=o(g(\alpha))$, if $\lvert f(\alpha)\rvert/\lvert g(\alpha)\rvert\to 0$
P_n, p	prolongation (cf. (5.2.2a), (5.3.3a))
$Q, Q_n, Q_{n,I}, Q_I, Q_{[a,b]}$	quadrature formula
$\mathbb{R}$	real numbers
R_n, r	restriction (cf. (5.2.2b), (5.3.3b))
sign(x)	sign of x with sign(0)=0
span{...}	space spanned by {...}
$w_i, w_{i,n}$	weights in the quadrature formula Q
X, Y	Banach spaces
X', Y'	dual spaces
X_n	subspace (cf. §4.4.1, §4.5.1)
$\mathbb{Z}$	integers
Γ	curve in $\mathbb{R}^2$ (or in $\mathbb{C}$) or surface in $\mathbb{R}^3$
δ_{ij}	Kronecker symbol: $\delta_{ij}=1$ for $i=j$, $\delta_{ij}=0$ otherwise
$\mu(A)$	(Lebesgue) measure of the set A
$\sigma(T)$	spectrum of the operator T (cf. §1.3.9)
Π_n	interpolation, projection
ω_d	surface of the d-dimensional unit sphere
Ω_-, Ω_+	interior and exterior domain (cf. Remark 7.1.11d)
$f(x+0), f(x-0)$	right-sided and left-sided limit
T'	dual operator of T (cf. §1.3.6)
T^*	adjoint operator of T
$\overline{\Omega}$	closure of the set Ω
$\partial\Omega$	boundary of the set Ω
$\frac{\partial}{\partial n}$	normal derivative (cf. (7.4.8b))
$⨍$	Cauchy principal value (cf. §7.1)
∇	gradient
Δ	Laplace operator (cf. §7.4)
$\lvert\cdot\rvert$	absolute value; in §§7-9 also Euclidean norm