Part I
Well-Posedness of Boundary-Value Problems

Introduction to Part I

Now let us explain the main purpose of the first part of the monograph. Earlier, many different sufficient conditions of unique solvability of the Cauchy problem for equation (1) have been obtained. In other words, one defined a certain space of initial data G, which depended on A and B (for instance: $G = (D(A) \cap D(B)) \times (D(A^2) \cap D(AB))$). Further, certain conditions on A and B were given. Then one showed that under these conditions on A and B, the Cauchy problem for (1) with initial conditions (i.c.) $y(0) = f_0$, $y'(0) = f_1$ is uniquely solvable for every $(f_0, f_1) \in G$.

Now, our main purpose is the following. We define a sufficiently wide space of boundary data G which depends on A and B. The question under study is: to find *all* equations (1) such that the Cauchy problem for (1) with boundary conditions (b.c.) $y(0) = f_0$, $y'(0) = f_1$ is uniquely solvable for every $(f_0, f_1) \in G$. In other words: to find a necessary and sufficient condition on A and B for equation (1) to have this property.

Such a question may be posed for other boundary-value problems as well. In the first part of the work, we answer this question for the Cauchy, Dirichlet, Neumann and the inverse Cauchy problems on a finite segment $[a, b]$, and for the Cauchy problem on a half-line $[a, +\infty)$ and the inverse Cauchy problem on a half-line $(-\infty, b]$ as well.

All these boundary-value problems are considered in the usual setting and in a «weak» setting as well.