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M.G. Krein's Lectures on Entire Operators

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Preface

This book is devoted to the theory of entire Hermitian operators, an important branch of functional analysis harmoniously combining the methods of operator theory and the theory of analytic functions. This theory anables various problems of classical and modern analysis to be looked at from a uniform point of view. In addition, it serves as a source for setting and solving many new problems in both theories. The three chapters of the book are based on the notes written by his students of M.G. Krein's lectures on the theory of entire operators with $(1,1)$-deficiency index which he delivered in 1961 at the Pedagogical Institute of Odessa, and on his works on the extension theory of Hermitian operators and the theory of analytic functions. The theory is further developed in the direction of solving the problems set up by Krein at ICM-66 in the first two appendices. The first concerns the case of Hermitian operators with arbitrary defect numbers, entire with respect to an ordinary gauge and to a generalized one as well. The other focuses on the entire operators representable by differential operators. The third appendix is the translation from Russian of the unpublished notes of Krein's lecture in which, in particular, the place of the theory of entire operators in the whole analysis is elucidated.

In Krein's mathematical heritage the theory of entire operators occupies a special position. From 1942 its intrinsic development as well as its connections with other fields of mathematics and applications were always subject of interest to him. In Krein's own reminiscences, the conceptual moments of this theory were drawn in his mind in the end of World War II. The changes in the character of the war after the Battle of Stalingrad (at the time he had been evacuated to Kuybyshev (now known as Samara)) revived his hope of coming back home to his beloved Odessa. This doubled his inspiration and by 1944 some articles had appeared in Dokl. Akad. Nauk SSSR, where the principal aspects of the theory of entire Hermitian operators whose deficiency index is $(1,1)$ were announced. This theory led him to adopt the same approach to investigate many classical problems of analysis, so different at first sight in their statements and the methods used to solve them. The power moment problem, the continuation problem for positive definite functions given on a finite interval and for spiral arcs, the description of all spectral functions of a string, the interpolation of functions and others are among them. In Krein's words, for any of these problems he saw a Hilbert space and a Hermitian operator on it hidden behind the scenes. It is to be noted that a number of new problems in the theory of analytic functions arose during the development of the theory of entire operators and its extension to more complicated situations.

In 1949 Krein developed the theory of entire operators with arbitrary finite defect numbers. This made it possible to solve the above problems in the matrix case. It should be noted that one would come across considerable analytical difficulties trying to deal with each problem separately. The case of infinite deficiency index had remained unsolved for a long time. This excited Krein because this was the case which allowed various partial differential equations problems to be considered. Krein emphasized this connection in his lectures and conversations with colleagues.

In 1966 a joint article by M. Krein and Sh. Saakyan devoted to entire operators with infinite defect numbers appeared in Dokl. Akad. Nauk SSSR. However, in the majority of problems which are simulated by differential equations, the associated operators are entire with respect to a generalized not ordinary gauge. This very fact probably motivated Krein in his one-hour lecture at the International Congress of Mathematicians (Moscow, 1966), to pay so much attention to the state of the general theory of entire operators and its relations to associated fields of mathematics, and to set the following three problems of great importance for the further development of this theory: 1) to investigate the case where a Hermitian operator is entire with respect to a gauge consisting of generalized elements; 2) to construct examples of entire operators generated by partial differential expressions; 3) to represent an arbitrary self-adjoint operator in the form of a differential one.

Unfortunately, Krein's results on entire operators have rarely been accessible for extensive study, for they are mostly announced, not published in detail. This situation troubled him greatly as the results were hardly known outside Odessa. Only his students and colleagues in Odessa were privileged to hear a detailed course of lectures on this topic. To publish it was Krein's dream. In due course the authors of this book agreed to prepare the lectures for publishing. By the end of the seventies the version adapted and extended according to Krein's suggestions had been prepared, but it was not published, because Krein became ill. The manuscript had been passed from hand to hand until in 1994 Prof. I. Gohberg proposed that we prepare it for publication. After certain modifications and additions it is presented in the form of this book. Krein's unpublished lectures on the theory of entire operators, whose deficiency index is $(1,1)$, and his paper in Ukr. Math. J. (1949), underlie all three chapters. The main concepts of this theory and its applications are given in accordance with these lectures. The missing proofs are taken from Krein's works on the operator theory and the theory of analytic functions.

There are three appendices. Appendix 1 covers the joint results by Krein and Saakyan on Hermitian operators with arbitrary defect numbers, entire with respect to an ordinary gauge, as well as the results by Yu. Shmulian concerning the case of a generalized gauge. Appendix 2 is devoted to solving problem 2). It is proved here that many operators representable in the form of a partial differential expression and corresponding boundary condition are entire. For the sake of simplicity of the exposition we consider differential expressions with unbounded operator coefficients. This allows us to cover a number of systems (finite and infinite) of ordinary differential equations and partial differential equations of hyperbolic type. Appendix 3 consists of the notes of Krein's talk at the Jubilee session of Moscow Mathematical Society (Moscow, 1964). It was written down by one of his students. The notes were taken from Krein's archive. Perhaps they do not reflect word by word all the nuances of that talk, but they give a vibrant idea of the theory of entire operators and its place in the general operator theory and its applications, and in paticular, in Krein's own research.

It should be noted that the book does not cover many other investigations by Krein , his students and followers, concerning the theory of entire operators and its applications. First of all, we would like to mention here the description of all self-adjoint extensions of an entire Hermitian operator whose distribution functions possess certain preassigned properties (for instance, the property of the support of a spectral function belonging to a certain set). We have not touched on the applications to the interpolation and extrapolation of functions at all, though they occupied a leading position in Krein's investigations during the last years of his life. Neither have we dwell on entire operators with a nondense domain. A partial discussion of the mentioned questions can be found in the survey by E. Tzekanovsky and Yu. Shmulian [1] and in the parer by V. Derkach and M. Malamud [1]. Our main purpose was to follow the spirit of Krein's unpublished lectures, and in doing so, to pay a tribute to one of the greatest mathematicians of the centure. He believed that the theory of entire operators would contribute significantly to the further development of analysis, once more proving its inherent unity.

Invaluable help in the preparation of the book was given us by Krein's students Professors Vadim Adamyan, Damir Arov and Adolf Nudelman (Odessa), to whom we are obliged for the sketches of his lectures . Professor Anatoly Kochubei (Kiev) was greatly assisted in improving the English version of the book. The text was composed on the computer given as a gift to the Institute of Mathematics (Kiev) by the brothers Myroslav and Lubomyr Prytulak (Canada). We are greatful to all of them. Certainly, the book could never have been published without Professor Israel Gohberg's (Israel) essential support. We are also thankful to him for his advice and useful remarks.