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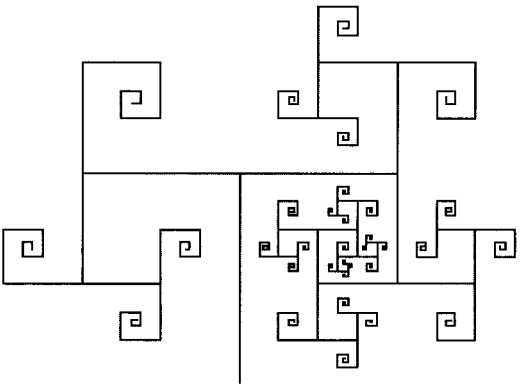
Mathematics and Computer Science II

Algorithms, Trees, Combinatorics and Probabilities

Brigitte Chauvin
Philippe Flajolet
Danièle Gardy
Abdelkader Mokkadem
Editors



Springer Basel AG

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2000 Mathematical Subject Classification 68M20, 68P30, 68Q25, 68Rxx, 68W20, 90B15

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Deutsche Bibliothek - Cataloging-in-Publication Data

Mathematics and computer science II: algorithms, trees, combinatorics and probabilities.
Brigitte Chauvin ... ed.. - Basel ; Boston ; Berlin : Birkhäuser, 2002
(Trends in mathematics)
ISBN 978-3-0348-9475-3 ISBN 978-3-0348-8211-8 (eBook)
DOI 10.1007/978-3-0348-8211-8

ISBN 978-3-0348-9475-3

The logo on the cover is a binary search tree in which the directions of child nodes alternate between horizontal and vertical, and the edge lengths decrease as 1 over the square root of 2. The tree is a Weyl tree, which means that it is a binary search tree constructed from a Weyl sequence, i.e., a sequence $(n\alpha) \bmod 1$, $n = 1,2,...$, where α is an irrational real number. The PostScript drawing was generated by Michel Dekking and Peter van der Wal from the Technical University of Delft.

Originally published by Birkhäuser Verlag, Basel - Boston - Berlin in 2002
Softcover reprint of the hardcover 1st edition 2002

ISBN 978-3-0348-9475-3

9 8 7 6 5 4 3 2 1 www.birkhauser-science.com

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Foreword

These are the Proceedings of the International Colloquium of Mathematics and Computer Science held at the University of Versailles-St-Quentin, September 18-20, 2002. This colloquium is the second one in a now regularly established series following the first venue in September 2000 in Versailles. The present issue is centered around Combinatorics, Random Graphs and Networks, Algorithms Analysis and Trees, Branching Processes and Trees, Applied Random Combinatorics.

The contributions have been carefully reviewed for their scientific quality and originality by the Scientific Committee chaired by P. Flajolet and composed by P. Chassaing, B. Chauvin , M. Drmota, J. Fill, P. Flajolet, A. Frieze, D. Gardy, S. Janson, C. Krattenthaler, G. Louchard, A. Mokkadem, R. Pemantle, P. Robert, J. Spencer , B. Ycart. We do thank them for their impressive work.

We also thank the invited speakers: D. Aldous, L. Devroye, S. Janson, M. Krivelevich, B. Pittel, H. Prodinger, M. Steele, the authors of submitted papers and the participants for their contribution to the success of the conference.

A. Baffert and C. Ducoin deserve special thanks for their kind and efficient contribution to the material preparation of the colloquium.

Finally, we express our acknowledgements to the laboratory of Mathematics (LAMA), the laboratory of Computer Science (PRISM), the University of Versailles-St-Quentin, the Centre National de la Recherche Scientifique (CNRS) and the Institut National de Recherche en Informatique et Automatique (INRIA) for providing generous financial and material support.

The Organizing Committee
B. Chauvin
S. Corteel
D. Gardy
J.F. Marckert
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B. Ycart

Preface

These colloquium proceedings address problems at the interface between mathematics and computer science, with special emphasis on discrete probabilistic models and their relation to algorithms. Combinatorial and probabilistic properties of random graphs and networks, random trees and branching processes, as well as random walks are central. Applications are to be found in analysis of algorithms and data structures, the major application field, but also in statistical theory, information theory, and mathematical logic. This colloquium is the second one in a now regularly established series, following the first venue in September 2000 in Versailles. The book features a collection of original refereed contributions supplemented by survey articles written by the invited speakers, L. Devroye, S. Janson, M. Krivelevich, B. Pittel, H. Prodinger, and M.Steele. As the field is burgeoning with applications at the frontier of several scientific disciplines, authors have been asked to provide a perspective on the various subfields involved.

Combinatorics. The starting point of many studies of random discrete models is combinatorics, which often provides us with exact representations in terms of counting generating functions. Arquès and Micheli develop the combinatorial enumerative theory of maps with special attention to colouring problems. Banderier presents a synthetic theory of walks with returns over the half integer line, which is motivated by fast random generation of combinatorial structures. Bousquet-Mélou shows us an explicitly solvable model of walks in the quarter-plane that originates with basic queueing theory questions and gives rise to elegant combinatorial developments. Brlek, Duchi, Pergola and Pinzani enrich the theory of "ECO-systems" now recognized to provide a unifying framework for many problems of combinatorial random generation. Krikun and Malyshev finely characterize the boundary of a random triangulation of the disk by means of combinatorial-analytic methods. Labelle, Lamathe, and Leroux successfully apply the theory of species to tree-like arrangements of cells and derive an original combination of explicit and asymptotic counting results.

Random Graphs and Networks. Following Erdős and Rényi's pioneering work around 1960, random graph models have been the subject of intense study for four decades. Baert, Ravelomanana, and Thimonier base a novel analysis of triangle free graphs on breadth-first search and its associated stochastic properties in the line of Spencer's approach. Chassaing and Schaeffer solve a long-standing open questions: What is the diameter of a random map? Their result is achieved by an examplary combination of bijective and probabilistic methods. Coppersmith, Gamarnik, and Sviridenko characterize the diameter of a random graph with long range interactions—such problems are of interest for percolation models but also in relation to the geometry of the web. Devroye, McDiarmid, and Reed analyse the emergence of giant components in two graph models that are similarly motivated by our desire to understand the "graph of the web". Krivelevich's invited lecture surveys random graph colouring: the problem is *NP*-complete in the worst-case, but the perspective changes dramatically when one switches from the pessimistic worst-case scenario to the more realistic average-case analysis. Le Bars demonstrates the usefulness of probabilistic inequalities in the analysis of some threshold phenomena of logic. Palaysi explores combinatorial and algorithmic aspects of

wavelength assignment in certain graphs representing interconnection networks. Last but not least, Steele's invited lecture reviews and revisits the celebrated problem of minimal spanning trees in graphs with random edge weights; his text gives for the first time surprisingly explicit formulæ out of which quantitative estimates can be derived.

Analysis of Algorithms and Trees. Trees are perhaps the most important structure of computer science. In particular, they appear as data structures in an amazing variety of domains, like textual data processing, data compression, fast retrieval of information, symbolic computation, and so on. Bourdon and Vallée exhibit versatile criteria informing us on conditions under which a complex pattern is or isn't likely to occur; their analysis is based on an original interplay of combinatorial and dynamical systems methods. The invited lecture by Devroye and Neininger develops an original analysis of a new structure, the suffix search tree that is a hybrid of two of the most important data structures, the suffix trie and the binary search tree. In particular, their study contributes significantly to our understanding of basic data structures when these are subjected to correlated data. Gittenberger proposes an approach via generating functions to the analysis of strata of nodes in random trees, which has applications to breadth-first search traversal. Hwang and Steyaert offer a definitive analytic treatment of the heap structure whose importance devolves from its widespread use in priority queue management and near-optimal sorting. Jacquet and Szpankowski are able to characterize the redundancy of Markov sources of order r by an ingenious combination of combinatorial and analytic methods. Quickfind, which is one of the most spectacular algorithms known for basic order statistics, is thoroughly analysed by Martínez, Panario, and Viola: their contribution even results in an eminently practical discussion of cut-off points for optimal performance. Nguyen-The's paper show that the study of random combinatorial trees is intimately related to the performance of basic algorithms of symbolic manipulation, in particular the formal simplification of expressions. The invited paper of Prodinger offers a unified analytic perspective on digit statistics in a wide variety of number representation systems. Beyond its pure number-theoretic aspects, this study finds numerous applications in parsing and compiling (register allocation), sorting networks, the design of adder circuits, mergesort, interpolation search, and even branching fractals.

Branching Processes and Trees. Branching processes constitute the probabilistic counterpart of the combinatorial theory of trees. Fayolle and Krikun provide ergodicity conditions for a model of random trees that is akin to binary search trees, but where evolution involves random insertions and deletions. The invited lecture of Janson tackles problems motivated by exhaustive search, along the lines of research by Ruskey and Knuth (see the forthcoming volume, *Combinatorial Algorithms*). In particular, Janson proves the existence of a limit distribution for the number of ideals in random trees and does so by a clever adaptation of contraction methods, using the size-biased branching process. Menshikov and Petritis report on recent results concerning random walks in a random environment on trees and their relationship to multiplicative chaos. Pittel's invited lecture revisits the loop erased random walk: this is a biased model of self-avoidance that is currently witnessing a number of spectacular developments. This article derives very precise estimates of limiting distributions that are also of interest for generating random spanning trees of the complete graph. Rösler, Topchii, and Vatutin finely characterize convergence rates of weighted branching processes and detect there

the occurrence of stable laws. Vatutin and Dyakonova develop informative limit theorems for a critical branching process in a random environment.

Applied random combinatorics. Random combinatorics interacts with many other areas of science. Huillet and Porzo re-examine a version of the parking problem that is known to have numerous applications in computer science (hashing algorithms, resource allocation), combinatorial optimization, as well as statistical mechanics and adsorption models. Mossel and O'Donnell examine the sensitivity to input noise of Boolean functions; their results have implications in learning theory, complexity theory, neural networks, and even (the authors argue) the American election system. Teytaud develops a set of new results in learning theory basing himself on ergodicity properties. Trouvé and Yu establish upper bounds on the number of questions a user asks in the case of hierarchically structured databases. Weiermann characterizes 0-1 laws in ordinal theory via analytic combinatorics; his paper also serves as a valuable introduction to random combinatorics in its relation to finite model theory and logic. The last contribution to this book, by Zhang and Golin, develops explicit formulæ for the number of spanning trees in structured graphs having a highly regular shape.

Altogether papers assembled in this volume offer snapshots of current research. At the same time, they illustrate the numerous ramifications of the theory of random discrete structures throughout mathematics and computer science. Many of them, in particular invited lectures, include carefully crafted surveys of their field. We thus hope that the book may serve both as a reference text and as a smooth introduction to many fascinating aspects of this melting pot of continuous and discrete mathematics.

Enjoy!

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Philippe Flajolet,
Danièle Gardy,
A. Mokkadem