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Matrix Norms and their Applications

Translated from the Russian
by A. Iacob

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PREFACE

A finite-dimensional linear topology admits infinitely many distinct geometric realizations, each obtained by choosing a particular norm. In the algebra of matrices it is natural to restrict oneselves to norms that possess the *ring property* $\|AB\| \leq \|A\| \|B\|$. If matrices are treated as linear operators in a linear space E, then upon endowing E with a norm one automatically endows the algebra of matrices with a norm. The matrix norms arising in this manner are called *operator* (alternatively, *induced* or *subordinate*) norms. For a certain period of time they constituted the only known class of matrix norms. Other examples were found only after in 1963 Yu. I. Lyubich (and independently, in 1964, J. Stoer) characterized the operator norms as minimal elements of the pointwise order structure on the set of all matrix norms. The indicated order has been subsequently subject to a detailed study by G. R. Belitskii. The most important result in this direction is the theorem asserting that all automorphisms of the order structure in question are in a certain sense inner. As a whole, a rather rich theory has been developed, an exposition of which is given in Chapters 3 and 4 of the monograph.

Chapter 1 has mainly a preparatory role. Its first two sections are purely introductory. However, beginning with §3, a number of relevant situations in which matrix norms are used are exhibited.

Chapter 2 makes a sufficiently thorough study of the boundary spectrum of contractions. It relies to a considerable extent to a combinatorial analysis that goes back to Frobenius, but has been elaborated in detail only after the publication of a note of Wielandt (1950) dedicated to Frobenius' centennary. A new direction emerged in works of Ptak and his collaborators, who introduced, and also computed in a number of instances the so-called critical exponents. This area is even today far from being studied exhaustively. In this monograph we indicate a number of other unsolved problems; among the solved ones there undoubtedly are some that can constitute a source of new problems.

We describe a variety of applications of matrix norms, not only because of their importance, but also to illustrate the principle of "fitting a norm to a given situation". This principle, which guides many applications of functional analysis, is particularly convincing in the finite-dimensional setting, where the choice of a norm is subject to no restrictions.

It is assumed that the reader is familiar with courses on linear algebra and calculus. Nevertheless, a number of facts from linear algebra are presented in order to make the exposition more accessible. With the more special aspects one can make acquaintance in the books recommended in the list of references. A number of brief comments on the literature are made at the end of the text. Therein we do not mention however the authors of sufficiently elementary or known theorems (except for those that usually bear the names of their authors).