

Bifurcation Problems and their Numerical Solution

ISNM 54
International Series of Numerical Mathematics
Internationale Schriftenreihe zur Numerischen Mathematik
Série internationale d'Analyse numérique

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P. Henrici, Zürich; H. O. Kreiss, Pasadena; A. Ostrowski, Montagnola, and J. Todd, Pasadena**

Bifurcation Problems and their Numerical Solution

Workshop on Bifurcation Problems and
their Numerical Solution
Dortmund, January 15-17, 1980

Edited by
H. D. Mittelmann and H. Weber,
Dortmund

1980



Springer Basel AG

**Library of Congress
Cataloging in Publication Data**

Workshop on Bifurcation Problems and their
Numerical Solution, University of Dortmund, 1980.
Bifurcation problems and their numerical solution.
(International series of numerical
mathematics ; v. 54)
English or German.
Bibliography: p.
includes index.
1. Differential equations, Nonlinear – Numerical
solutions – Congresses. 2. Differential equations,
Partial – Numerical solutions – Congresses.
3. Bifurcation theory – Congresses
I. Mittelman, H. D., 1945 – II. Weber, H., 1948 –
III. Title IV. Series
QA370.W67 1980 515.3'55 80 – 18512

**CIP-Kurztitelaufnahme
der Deutschen Bibliothek**

**Bifurcation problems and their numerical
solution** / Workshop on Bifurcation
Problems and their Numer. Solution,
Dortmund, January 15–17, 1980.
Ed. by H. D. Mittelman and H. Weber. –
Basel, Boston, Stuttgart : Birkhäuser, 1980.
(International series of numerical
mathematics : 54)

NE: Mittelman, Hans Detlef [Hrsg.];
Workshop on Bifurcation Problems and
their Numerical Solution
(1980, Dortmund)

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Originally published by Birkhäuser Verlag Basel in 1980.
ISBN 978-3-7643-1204-6
ISBN 978-3-0348-6294-3 (eBook)
DOI 10.1007/978-3-0348-6294-3

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List of Participants

- Dr. *W.-J. Beyn*, Universität Konstanz, Fachbereich Mathematik,
Postfach 7733, D-7750 Konstanz (FRG)
- Prof. Dr. *R. Böhme*, Ruhr-Universität Bochum, Abteilung für Mathematik,
Universitätsstraße 150, D-4630 Bochum-Querenburg (FRG)
- Prof. Dr. *E. Bohl*, Universität Konstanz, Fachbereich Mathematik,
Postfach 7733, D-7750 Konstanz (FRG)
- Dr. *A. Bongers*, Universität Mainz, Fachbereich Mathematik,
Saarstraße 21, D-6500 Mainz (FRG)
- Prof. Dr. *R. D. Grigorieff*, Technische Universität Berlin, Fachbereich Mathematik,
Straße des 17. Juni 135, D-1000 Berlin (FRG)
- Dr. *G. Heinemann*, GHS Kassel, Fachbereich 17,
Heinrich-Plett-Straße 40, D-3500 Kassel
- Prof. Dr. *K. Kirchgässner*, Universität Stuttgart, Mathematisches Institut B,
Pfaffenwaldring 57, D-7000 Stuttgart 80 (FRG)
- Dr. *T. Küpper*, Universität Köln, Mathematisches Institut,
Weyertal 86-90, D-5000 Köln 41 (FRG)
- Prof. Dr. *W. F. Langford*, McGill University, Department of Mathematics,
Montreal, Que. H3A 2K6 (Canada)
- Prof. Dr. *H. D. Mittelmann*, Universität Dortmund, Abteilung Mathematik,
Postfach 50 05 00, D-4600 Dortmund 50 (FRG)
- Dr. *G. Moore*, University of Bath, School of Mathematics,
Claverton Down BA2 7AY (U. K.)
- Prof. Dr. *G. H. Pimbley*, University of California, Los Alamos Scientific Laboratory,
Los Alamos, NM 87544 (USA)
- Dr. *J. Scheurle*, Universität Stuttgart, Mathematisches Institut B,
Pfaffenwaldring 57, D-7000 Stuttgart 80 (FRG)
- Dr. *R. Scholz*, Universität Freiburg, Institut für Angewandte Mathematik,
Hermann Herder-Straße 10, D-7800 Freiburg (FRG)
- Dr. *R. Seydel*, TU München, Fachbereich Mathematik,
Arcisstraße 21, D-8000 München 2 (FRG)
- Dr. *D. Socolescu*, Universität Karlsruhe, Institut für Angewandte Mathematik,
Englerstraße, D-7500 Karlsruhe (FRG)
- Prof. Dr. *A. Spence*, University of Bath, School of Mathematics,
Claverton Down BA2 7AY (U. K.)
- Dr. *J. Sprekels*, Universität Hamburg, Institut für Angewandte Mathematik,
Bundesstraße 55, D-2000 Hamburg 13 (FRG)
- Prof. Dr. *U. Staude*, Universität Mainz, Fachbereich Mathematik,
Saarstraße 21, D-6500 Mainz (FRG)
- Dr. *P. Vielsack*, Universität Karlsruhe, Institut für Mechanik,
D-7500 Karlsruhe (FRG)
- Dr. *H. Voß*, GHS Essen, Fachbereich 6 – Mathematik,
Universitätsstraße 2, D-4300 Essen (FRG)
- Dr. *H. Weber*, Universität Dortmund, Abteilung Mathematik,
Postfach 50 05 00, D-4600 Dortmund 50 (FRG)
- Prof. Dr. *B. Werner*, Universität Hamburg, Institut für Angewandte Mathematik,
Bundesstraße 55, D-2000 Hamburg 13 (FRG)
- Dipl.-Math. *U. Wiesweg*, Universität Dortmund, Abteilung Wirtschaftswissenschaften,
Lehrstuhl für Betriebsinformatik, Postfach 50 05 00, D-4600 Dortmund 50 (FRG)

Preface

The ›Workshop on Bifurcation Problems and their Numerical Solution‹, organized by the editors of this volume, took place at the University of Dortmund from January 15–17, 1980. The main object of this meeting was to bring together mathematicians from different places who were working in this field. Seventeen lectures were presented and discussed at the workshop. Most of them are published in these proceedings together with a survey article on the whole subject.

The contributions cover various aspects, ranging from new results in the Ljusternik-Schnirelman theory to numerical methods for the computation of branch points, and from the discretization by multiple shooting to new results about bifurcation mode interactions. Further topics include the non-occurrence of bifurcation for discretized problems and bounds for turning points of nonlinear eigenvalue problems.

The organizers would like to express their gratitude to all participants of the workshop, to the authors of the published contributions and especially to the editors of the ISNM-series as well as Birkhäuser Verlag for the rapid publication of these proceedings. Financial support by the ›Minister für Wissenschaft und Forschung des Landes Nordrhein-Westfalen‹ and by the ›Gesellschaft der Freunde der Universität Dortmund‹ is gratefully acknowledged.

Dortmund, May 1980

H. D. Mittelmann
H. Weber