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# Advances in Applied Analysis

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# Preface

In January 2010, the Council of Young Scientists of the Belarusian State University organized the 3rd International Winter School “Modern Problems of Mathematics and Mechanics”. Young researchers, graduate, master and post-graduate students from Belarus, Lithuania, Poland and Ukraine participated in this school. They attended lectures of well-known experts in Analysis and its Applications. Six cycles of 3–4 lectures each were presented by Dr. V. Kisil (Leeds University, UK), by Prof. A. Laurinčikas (Vilnius University, Lithuania), Prof. Yu. Luchko (Beuth Technical University of Applied Sciences, Berlin, Germany), Prof. V. Mityushev (Krakow Pedagogical Academy, Poland), Prof. S. Plaksa (Institute of Mathematics, National Academy of Sciences, Ukraine) and Dr. S. Rogosin (Belarusian State University, Minsk, Belarus).

The book is made up of extended texts of the lectures presented at the School. These lectures are devoted to different problems of modern analysis and its applications. Below we briefly outline the main ideas of the lectures. Since they have an advanced character, the authors tried to make them self-contained.

A cycle of lectures by Dr. V. Kisil “Erlangen Program at Large: An Overview” describes a bridge between modern analysis and algebra. The author introduces objects and properties that are invariant under a group action. He begins with conformal geometry and develops a special functional calculus. He uses, as a characteristic example, a construction of wavelets based on certain algebraic techniques.

Prof. A. Laurinčikas deals with the notion of universality of functions. His cycle is called “The Riemann zeta-function: approximation of analytic functions”. He shows that one of the best examples of universality is the classical Riemann zeta-function. So this lecture can be considered as describing the connection between Analysis and Number Theory.

A cycle of lectures by Prof. Yu. Luchko “Anomalous diffusion: models, their analysis, and interpretation” presents a model of anomalous diffusion. This model is given in terms of differential equations of a fractional order. The obtained equations and their generalizations are analyzed with the help of both the Laplace-Fourier transforms (the Cauchy problems) and the spectral method (initial-boundary-value problems).

Prof. V. Mityushev presents in his cycle “R-linear and Riemann–Hilbert problems for multiply connected domains” elements of constructive analysis related to the solution of boundary value problems for analytic functions. He pays partic-

ular attention to further application of the obtained results in the theory of 2D composite materials and porous media.

Another type of applications are presented in the cycle of lectures by Prof. S. Plaksa “Commutative algebras associated with classic equations of mathematical physics”. In his work he develops a technique for application of the theory of monogenic functions in modern problems of mathematical physics. In particular, he studies axial-symmetric problems of the mechanics of continuous media.

Dr. S. Rogosin describes some modern ideas that can be applied to the study of certain free boundary problems (“2D free boundary value problems”). In particular, he develops an illustrative example dealing with so-called Hele-Shaw boundary value problem. This problem is reduced to a couple of problems, namely, an abstract Cauchy–Kovalevsky problem and a Riemann–Hilbert–Poincaré problem for analytic functions.

The book is addressed to young researchers in Mathematics and Mechanics. It can also be used as the base for a course of lectures for master-students.