Operator Theory: Advances and Applications
Volume 212

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Elementary Operators and Their Applications

3rd International Workshop held at Queen’s University Belfast, 14–17 April 2009

Raúl E. Curto
Martin Mathieu
Editors
## Contents

Introduction ............................................................. vii  
Picture of Participants ................................................ viii  
List of Participants .................................................... ix  
Programme .............................................................. x  
Abstracts of Talks ...................................................... xi  

### Contributions

**N. Boudi and M. Mathieu**  
Elementary Operators that are Spectrally Bounded .................. 1  

**D. Kitson**  
The Browder Spectrum of an Elementary Operator .................... 17  

**B. Magajna**  
Approximation of Maps on $C^*$-Algebras by Completely  
Contractive Elementary Operators .................................. 25  

**P. Rosenthal**  
Some Not-quite-elementary Operators ................................. 41  

**V.S. Shulman and Yu.V. Turovskii**  
Topological Radicals, II. Applications to Spectral Theory of  
Multiplication Operators ............................................ 45  

**V.S. Shulman and L. Turowska**  
An Elementary Approach to Elementary Operators on $\mathcal{B}(H)$ ...... 115  

**R.M. Timoney**  
Computation Versus Formulae for Norms of  
Elementary Operators .................................................. 133  

Open Problems ........................................................... 151
Introduction

The Third International Workshop on Elementary Operators and Their Applications was held at the Department of Pure Mathematics of Queen’s University Belfast between April 14 and 17, 2009. It was organised by Dr Martin Mathieu as a satellite to the joint meeting of the British Mathematical Colloquium and the Annual Meeting of the Irish Mathematical Society that took place at NUI Galway the week before. The funding received from the Irish Mathematical Society and the London Mathematical Society is gratefully acknowledged.

This series of workshops started in June 1991 at the Heinrich Fabri-Institute of the University of Tübingen, Germany and was continued in September 2001 at the University of Helsinki, Finland. At each of these meetings substantial progress in the research on elementary operators was reported. The proceedings volume of the 1991 workshop listed a number of difficult problems some of which, notably the norm problem for elementary operators have been solved in the meantime.

The present volume once again aims to present the state-of-the-art of our understanding of the theory of elementary operators and their applications. In fact, the applications are far more wide-ranging than one might expect and touch on many areas in Mathematics but also in Physics, such as Solid State Physics and Quantum Information Theory. Elementary operators are so simple in their definition that they occur everywhere; for instance, every linear mapping on a finite-dimensional semisimple Banach algebra is an elementary operator. It is no surprise therefore that one tries to approximate various classes of operators in infinite dimensions by elementary operators; some of the contributions in this book discuss these.

Yet the simple definition of an elementary operator entirely conceals the intricate and often challenging interplay between structural properties of elementary operators and the underlying algebras. This volume contains solicited articles by speakers at the workshop ranging from expository surveys to original research papers, each of which carefully refereed. They all bear witness to the very rich mathematics that is connected with the study of elementary operators, may it be multivariable spectral theory, the invariant subspace problem or tensor products of C*-algebras.
As always in mathematics, results lead to new questions, and therefore the volume concludes with another list of open problems some of which were explained in the workshop problem session. May they inspire further exciting insights on elementary operators!

31 July 2010

Raúl E Curto, Iowa City
Martin Mathieu, Belfast

A picture of the workshop participants taken by the organiser
List of Participants

Jerónimo Alaminos, Universidad de Granada, Granada, Spain
Rob Archbold, University of Aberdeen, Aberdeen, Scotland
Ariel Blanco, Queen’s University Belfast, Belfast, Northern Ireland
Nadia Boudi, Université Moulay Ismail, Meknès, Morocco
Raúl Curto, University of Iowa, Iowa City, USA
Lawrence Fialkow, SUNY at New Paltz, New Paltz, USA
Robin Harte, Trinity College Dublin, Dublin, Ireland
Derek Kitson, Trinity College Dublin, Dublin, Ireland
Rupert Levene, Trinity College Dublin, Dublin, Ireland
Bojan Magajna, University of Ljubljana, Ljubljana, Slovenia
Martin Mathieu, Queen’s University Belfast, Belfast, Northern Ireland
Martin McGarvey, Queen’s University Belfast, Belfast, Northern Ireland
F. Javier Meri, Granada, Universidad de Granada, Granada, Spain
Savvas Papapanagides, Queen’s University Belfast, Belfast, Northern Ireland
Robert Pluta, Trinity College Dublin, Dublin, Ireland
Peter Rosenthal, University of Toronto, Toronto, Canada
Eero Saksman, University of Helsinki, Helsinki, Finland
Victor Shulman, Vologda State Technical University, Vologda, Russia
Franek Szafraniec, Jagiellonian University, Kraków, Poland
Richard Timoney, Trinity College Dublin, Dublin, Ireland
Ivan Todorov, Queen’s University Belfast, Belfast, Northern Ireland
Ta Ngoc Tri, Lancaster University, Lancaster, England
Aleksej Turnšek, University of Ljubljana, Ljubljana, Slovenia
Lyudmila Turowska, Chalmers University of Technology and the University of Gothenburg, Gothenburg, Sweden
Hans-Olav Tylli, University of Helsinki, Helsinki, Finland
Armando R. Villena, Universidad de Granada, Granada, Spain
<table>
<thead>
<tr>
<th>Time Slot</th>
<th>Tuesday, 14 April</th>
<th>Wednesday, 15 April</th>
<th>Thursday, 16 April</th>
<th>Friday, 17 April</th>
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<tbody>
<tr>
<td>09:30 – 10:20</td>
<td>Rob Archbold</td>
<td>Bojan Magajna</td>
<td>Lyudmila Turowska</td>
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<td>10:30 – 11:00</td>
<td>Robin Harte</td>
<td>Martin McGarvey</td>
<td>Problem Session</td>
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<td>11:00 – 11:30</td>
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<td></td>
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<td>11:30 – 12:20</td>
<td>Richard Timoney</td>
<td>Nadia Boudi</td>
<td>Hans-Olav Tylli</td>
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<tr>
<td>12:30 – 14:00</td>
<td>Registration</td>
<td>Lunch Break</td>
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<tr>
<td>14:00 – 14:50</td>
<td>Welcome</td>
<td>Raúl Curto</td>
<td>Larry Fialkow</td>
<td>End of workshop</td>
</tr>
<tr>
<td>15:00 – 15:30</td>
<td>Ivan Todorov</td>
<td>Franek Szafraniec</td>
<td>Derek Kitson</td>
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<td>16:00 – 16:50</td>
<td>Ariel Blanco</td>
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<td>Aleksej Turnsek</td>
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<td>17:00 – 17:50</td>
<td>Peter Rosenthal</td>
<td>Football Match</td>
<td></td>
<td>Eero Saksman</td>
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<td>18:00 – 19:00</td>
<td>Reception</td>
<td>Queen’s PEC</td>
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*Note: The Conference Dinner includes a Football Match or a Black Taxi Tour through Belfast.*
States, representations and norms of elementary operators
Rob Archbold
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Coauthors: Douglas Somerset (Aberdeen) and Richard Timoney (Dublin)

We discuss recent work with Douglas Somerset and Richard Timoney which
aims to localize the connection between matricial norms of elementary operators
on a $C^*$-algebra and the weak*-approximation of factorial states.
Let $T$ be an elementary operator on a $C^*$-algebra $A$, $\pi$ an irreducible rep-
resentation of $A$ and $T^\pi$ the induced operator on $\pi(A)$. It is automatic that if $n$
is a positive integer then $\|T^\pi\|_n \leq \|T\|_n$ and that the $n$-positivity of $T$ implies
the $n$-positivity of $T^\pi$. We show that if the upper multiplicity $M_U(\pi) > 1$
then $\|T^\pi\|_k \leq \|T\|_n$ for certain values of $k > n$ and that the $n$-positivity of $T$
implies the $k$-positivity of $T^\pi$ for these same larger values of $k$.
The results are obtained by using Timoney’s descriptions of matricial norms
and $k$-positivity in terms of the “tracial geometric mean” and factorial states. The
condition on $M_U(\pi)$ allows one to approximate factorial states associated with $\pi$
by type I factorial states of lower degree.

These localizations at irreducible representations $\pi$ lead to new proofs of
various characterizations of the class of antiliminal-by-abelian $C^*$-algebras in terms
of factorial states and elementary operators.

On the cohomology of Banach operator algebras
Ariel Blanco
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We present new results on the bounded cohomology of Banach operator al-
gebras. Some of them will be generalizations of well-known results for properly
infinite $C^*$-algebras.
On spectrally bounded elementary operators
Nadia Boudi
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I will first recall basic results on spectrally bounded linear maps, and then I will describe some spectrally bounded elementary operators. At the end of my talk, I will discuss some related conditions.

2-variable weighted shifts built from elementary tensors
Raúl E. Curto
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Coauthors: Sang Hoon Lee and Jasang Yoon

We consider the class $TC$ of 2-variable weighted shifts with tensor core. These are shifts whose restrictions to a large invariant subspace split as

$$(I \otimes W_\alpha, W_\beta \otimes I),$$

so these restrictions are canonically associated to left and right multiplication operators.

For the class $TC$, we study the Reconstruction-of-the-Measure Problem (ROMP), which consists of finding necessary and sufficient conditions on a pair $T = (T_1, T_2) \in TC$ that guarantee its subnormality. ROMP is intimately connected to the Lifting Problem for Commuting Subnormals (LPCS), which asks for necessary and sufficient conditions for a pair of subnormal operators on Hilbert space to admit commuting normal extensions. It is well known that the commutativity of the pair is necessary but not sufficient, and it has recently been shown that the joint hyponormality of the pair is necessary but not sufficient. Moreover, while abstract solutions of LPCS exist, concrete solutions are only known in very specific situations.

Our previous research has shown that many of the nontrivial aspects of LPCS are best detected within the class $H_1$ of commuting hyponormal pairs of subnormal operators, so we focus our attention on this class. $TC$ is a large subclass of $H_1$, and we have been able to provide a complete solution of LPCS within $TC$. We also show that for a pair $T \in TC$, the subnormality of $(T_1^2, T_2)$ (typically easier to verify) automatically implies the subnormality of $T$.

Abstract vs. concrete solutions to multivariable truncated moment problems
Lawrence Fialkow
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In previous work with R.E. Curto, we formulated two solutions to the truncated multivariable moment problem, one in terms of “flat extensions” of positive moment matrices, another in terms of positive extensions of linear functionals on
polynomial spaces associated with the moment data. These solutions are difficult to implement concretely. We discuss some recent concrete results concerning multivariable quadratic moment problems and the bivariatic quartic moment problem (joint work with Jiawang Nie), and a concrete solution to the $y = x^3$ truncated moment problem.

**Hermitian subspaces and Fuglede operators (Contributed talk)**
Robin Harte
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A real-linear subspace $H \subseteq A$ of a complex linear algebra with identity 1 will be called a “Hermitian subspace” if it satisfies

$$H \cap iH = \{0\} ; \ 1 \in H .$$

There is induced an involution $*: h + ik \mapsto h - ik$ on the complex linear subspace $H + iH \subseteq A$. When $A = B(X)$ is the bounded linear operators on a Banach space an involution on $H + iH$ gives rise to “Fuglede” operators $T \in H + iH$ for which

$$T^{-1}(0) \subseteq T^*{-1}(0) .$$

There is also induced another involution on an appropriate subspace of the “elementary operators” on $A = B(X)$. In this discussion we relate the Fuglede property for $T$ and $S^*$ to Fuglede properties for $L_T - R_S$ and $L_T R_S$.

**A multivariable spectral mapping theorem (Contributed talk)**
Derek Kitson
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The classical notions of ascent and descent for a linear operator on a vector space can be extended to arbitrary collections of operators. Using this fact we construct a Browder joint spectrum for commuting $n$-tuples of bounded operators on a complex Banach space. This Browder joint spectrum is compact valued and satisfies a multivariable spectral mapping theorem. Connections to the Browder spectrum of an elementary operator will be discussed.

**Approximation by elementary operators**
Bojan Magajna
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On which $C^*$-algebras $A$ can all complete contractions, that preserve closed two-sided ideals, be approximated by completely contractive elementary operators? In the operator norm topology this is possible only if $A$ is a finite direct sum of homogeneous $C^*$-algebras arising from $C^*$-bundles of finite type. On the other hand, pointwise approximation is always possible if $A$ is nuclear, but the precise characterization is still open.
Normalisers, nest algebras and tensor products (Contributed talk)
Martin McGarvey
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If $A$ is an operator algebra acting on a Hilbert space $H$, a normaliser of $A$ is an operator $T$ on $H$ such that $T^*AT \subseteq A$ and $TAT^* \subseteq A$. The set of all normalisers of $A$ is denoted by $N(A)$.

We will show that if $A$ is the tensor product of finitely many continuous nest algebras, $B$ is a CDCSL algebra and $N(A) = N(B)$ then either $A = B$ or $A = B^*$.

Some not-quite-elementary operators
Peter Rosenthal
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Coauthors: Don Hadwin and Eric Nordgren

This will be a discussion of joint work with Don Hadwin and Eric Nordgren concerning operator equations such as $AXB + CYD = Z$.

Alexandrov measures and connectedness in the space of composition operators
Eero Saksman
*University of Helsinki*
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Coauthors: Eva Gallardo (Zaragoza), Maria Gonzales (Cadiz) and Pekka Nieminen (Helsinki)

We show that compact composition operators on the Hardy space $H^2$ do not form a component in the norm topology of operators. This answers a question of J.H. Shapiro and C. Sundberg from the early 1990’s.

On elementary operators with compact coefficients
Victor Shulman
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Coauthors: Yuri Turovskii

W. Wojtynsky proved in 1970 that each solution of a linear operator equation of the form $\sum_k A_kXB_k + AX + XB = \lambda X$, where $A_k, B_k, A, B$ are compact operators on a Banach space and $\lambda \neq 0$, is a nuclear operator. We suggest a new approach to this result and extend it considering eigenvectors and spectral subspaces of elementary operators with sufficiently many compact coefficients and of operators of more general form (in particular infinite sums are admitted).


On Murphy’s positive definite kernels and Hilbert $C^*$-modules
(Contributed talk)
Franciszek Hugon Szafraniec
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The paper the title refers to is that in Proceedings of the Edinburgh Mathematical Society, 40 (1997), 367–374. Taking it as an excuse we intend to realize a twofold purpose: 1. to atomize that important result showing by the way connections which are out of favour; 2. to rectify a tiny piece of history. The objective 1 is going to be achieved by adopting means adequate to goals; it is of great gravity and this is just Mathematics. The other, 2, comes from the author’s internal need of showing how ethical values in Mathematics are getting depreciated. The latter have nothing to do with the previous issue; the coincidence is totally accidental.

Computation versus formulae for norms of elementary operators
Richard Timoney
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We survey long standing and newer results on norms of elementary operators, including those of Stampfli (on derivations) and Haagerup (on completely bounded norms). We consider them from the point of view of their effectiveness for a practical problem and their value as a theoretical device. We discuss some aspects where there may be scope for further progress.

$s$-numbers of elementary operators (Contributed talk)
Ivan Todorov
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Coauthors: M. Anoussis and V. Felouzis

A theorem of Fong and Sourour states that an elementary operator acting on $B(H)$ is compact if and only if it has a representation where its symbols are compact operators. In this talk, a quantitative version of this result will be presented where the behaviour of the $s$-numbers of an elementary operator is linked to the behaviour of the singular numbers of its symbols.

Orthogonality and Fuglede–Putnam theorem
Aleksej Turnsek
University of Ljubljana
aleksej.turnsek@fmf.uni-lj.si

We present some results on orthogonality of the range and the kernel of elementary operators and connect them with the Fuglede–Putnam theorem.
Beurling–Pollard type theorems
Lyudmila Turowska
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Coauthors: Victor Shulman

We establish a version of Beurling–Pollard theorem for operator synthesis and apply it to derive some results on linear operator equations. In addition, we establish a Beurling–Pollard theorem for weighted Fourier algebra and use it to obtain ascent estimate for operators that are functions of generalized scalar operators. This is a joint work with Victor Shulman.

Two-sided multiplication operators on $L(L^p)$
Hans-Olav Tylli
University of Helsinki
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I will discuss the following results from [LST] and [JS] about the qualitative properties of the basic multiplication operators $L_AR_B$, where $S \mapsto ASB$ for fixed operators $A, B \in \mathcal{L}(L^p)$, on the space $\mathcal{L}(L^p)$ of the bounded operators on $L^p$.

(i) $L_AR_B$ is strictly singular $\mathcal{L}(L^p) \rightarrow \mathcal{L}(L^p)$ for $1 \leq p \leq \infty$ if and only if $A \neq 0$ and $B \neq 0$ are strictly singular operators on $L^p$, [LST],

(ii) $L_AR_B$ is weakly compact $\mathcal{L}(L^p) \rightarrow \mathcal{L}(L^p)$ for $2 < p < \infty$ if and only if either $A$ is compact, $B$ is compact, or $JA \in G_{\ell^2}$ and $B \in G_{\ell^p}$ for some isometry $J : L^p \rightarrow L^\infty$, [JS]. Here $A \in \mathcal{L}(L^p)$ belongs to the factorization ideal $G_{\ell^p}$ if $A = UV$, where $U \in \mathcal{L}(\ell^r, L^p)$ and $V \in \mathcal{L}(L^p, \ell^r)$.

References
