

Operator Theory: Advances and Applications

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A Glimpse at Hilbert Space Operators

Paul R. Halmos in Memoriam

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Preface

Paul Richard Halmos, who lived a life of unbounded devotion to mathematics and to the mathematical community, died at the age of 90 on October 2, 2006. This volume is a memorial to Paul by operator theorists he inspired.

Paul's initial research, beginning with his 1938 Ph.D. thesis at the University of Illinois under Joseph Doob, was in probability, ergodic theory, and measure theory. A shift occurred in the 1950s when Paul's interest in foundations led him to invent a subject he termed algebraic logic, resulting in a succession of papers on that subject appearing between 1954 and 1961, and the book *Algebraic Logic*, published in 1962.

Paul's first two papers in pure operator theory appeared in 1950. After 1960 Paul's research focused on Hilbert space operators, a subject he viewed as encompassing finite-dimensional linear algebra.

Beyond his research, Paul contributed to mathematics and to its community in manifold ways: as a renowned expositor, as an innovative teacher, as a tireless editor, and through unstinting service to the American Mathematical Society and to the Mathematical Association of America. Much of Paul's influence flowed at a personal level. Paul had a genuine, uncalculating interest in people; he developed an enormous number of friendships over the years, both with mathematicians and with nonmathematicians. Many of his mathematical friends, including the editors of this volume, while absorbing abundant quantities of mathematics at Paul's knee, learned from his advice and his example what it means to be a mathematician.

The first section of this volume contains three tributes to Paul written on the occasion of his death. They elaborate on the brief remarks in the preceding paragraph, and are reproduced here with the kind permission of their authors and their original publishers. The last item in the first section reproduces the late George Piranian's Mathematical Review of Paul's article *How to write mathematics*. A list of Paul's publications comprises the next section, which is followed by a section of photographs of Paul and photographs taken by Paul.

The main and final section consists of a collection of expository articles by prominent operator theorists. From these articles, this generation of operator theorists and future generations will get a glimpse of many aspects of their subject, and of how Paul enriched and advanced it through his fundamental insights and prescient questions.

Acknowledgement

We thank the authors of the expository articles for the high quality of their expositions.

We thank Thomas Hempfling, Birkhäuser's Executive Editor for Mathematics, for his constant support and sound advice.

We thank Mary Jennings for volunteering her expertise in assembling the photo section. This was an arduous task that Mary carried out with painstaking care, offering helpful suggestions along the way. (Other photo acknowledgements are in the introduction to the photo section.)

This volume was first proposed by the late Israel Gohberg, the founder and until his recent death the chief editor of the series in which it appears. With Israel's passing, operator theory has lost another of its giants.

Sheldon Axler, Peter Rosenthal, Donald Sarason

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