

The First Five Years of Teaching Mathematics (FIRSTMATH)

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The First Five Years of Teaching Mathematics (FIRSTMATH)

Concepts, Methods and Strategies
for Comparative International Research

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Foreword

This book reports the details of a careful inquiry into what it would take to conduct a scientifically rigorous study that would allow us to answer a number of intriguing and important questions: *How do beginning teachers of mathematics in primary and secondary schools differ nationally and cross-nationally in their demographic characteristics, preparation, knowledge for teaching, teaching assignments, working conditions, teaching practice, and pupil learning?* Readers of this book will benefit from the proof-of-concept foundational work on conceptualization and methodology that the authors have conducted. To understand why it would be critical to conduct such an investigation, I think it is useful to step back and consider a few fundamental observations.

Mathematics is a staple of schooling throughout the world Though countries across the globe vary with regard to dominant languages, historical traditions, and cultural practices, they all share at least one important commonality: in every country, at every grade level, on every day during the academic year, a student attending school is likely to receive mathematics instruction. Moreover, among all the school subjects that a student is expected to learn, mathematics is generally viewed as the one most likely to be contingent on what is learned in school. Whereas students may have many opportunities to sharpen literacy skills or to learn history or science independent of classroom instruction, it is generally acknowledged that the learning of mathematics depends to a great extent on the quantity and quality of classroom instruction. Thus, educators, public policy professionals, and the general public in most countries are very interested in promoting effective mathematics teaching in schools.

Student attainment of mathematical proficiency varies among countries Over the past 50 years, international surveys of the mathematics achievement of students around the world, such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), have reported considerable differences in performance across participating countries. The observation of such variation over time has given rise to a search for explanations.

Though those who attempt to compare education in different countries face a daunting task aptly characterized by Torsten Husén (1983) as “comparing the incomparable,” the search has been vigorous. Many factors have been examined and proposed as possible explanations for performance variation, including the language of instruction, a host of affective and cognitive factors, cultural influences, and various technical details of schooling such as school, classroom, and lesson organization (Clarke, 2003). With regard to variation in mathematics performance, curriculum has been arguably the most extensively examined and compared educational feature (e.g., Schmidt et al., 2001; Travers & Westbury, 1989).

Teachers are central agents in the enacted mathematics curriculum Over three decades ago, Travers and Westbury (1989) suggested an important distinction between *curriculum as intended* and *curriculum as enacted*. In the Second International Mathematics Study (SIMS), the intended curriculum (as represented in official documents and textbooks) was contrasted with the implemented curriculum (measured through questionnaires given to teachers). These curriculums were, in turn, related to the *attained curriculum*, as reflected in students’ performance on SIMS test items. The distinction offered by Travers and Westbury suggests both the promise and peril of satisfying one’s hopes for understanding critical aspects of mathematics teaching and learning in a country through an analysis of intended curriculum alone. Though official curriculum documents and widely used textbooks serve as blueprints for school mathematics and can serve as tools to influence classroom instruction, they do not completely determine the content and conduct of classroom instruction (i.e., the enacted curriculum).

The enactment of curricular resources lies at the heart of the work of classroom teachers. Mathematics teachers design lessons, select tasks from curricular resources, and conduct lessons in interaction with students. Understanding variation in students’ mathematical proficiency across the world requires that we attend not only to variability among countries in the expectations for students in the mathematics curriculum but also to variations among mathematics teachers and mathematics teaching. Some investigations, patterned after the pioneering TIMSS analysis of videotaped classroom instruction from three participating education systems (Germany, Japan, and the United States) (Kawanaka, Stigler & Hiebert, 1999), have examined samples of actual classroom instruction with a focus on the teaching or learning activity in the classroom (e.g., Clark, Keitel, & Shimizu, 2006). These studies have shown both the centrality of the work of teachers in shaping classroom instruction and students’ learning opportunities and the existence of differences across countries in the nature of the work of mathematics teachers.

How teachers are prepared to teach mathematics matters Variations across countries in the preparation of mathematics teachers were carefully examined in the Teacher Education and Development Study in Mathematics (TEDS-M)—a seminal investigation of mathematics teacher preparation programs across the world (Tatto, Schwillie, Senk, Ingvarson, Peck, & Rowley, 2008; Tatto, 2013). Involving 17 different countries and nearly 25,000 prospective teachers, TEDS-M was the first

large-scale international comparative study of the outcomes of teacher education that teacher educators designed and implemented.

Several important lessons were learned from TEDS-M. In particular, wide variability within and across countries was found in both the content and pedagogical knowledge of teacher education graduates and in the conditions and opportunities to learn provided to future teachers in their academic programs and field experiences. TEDS-M also underscored the difficulty of attracting academically strong students to the teaching profession, especially given conditions in the labor market in some countries.

Beginning teaching is both an extension of teacher preparation and the initiation of new professional learning The TEDS-M study found that substantial between-country differences exist in the preparation of future primary and secondary school teachers of mathematics (Tatto, 2018a, b). In some countries, for example, future teachers are extensively prepared in mathematics content but receive little training in mathematics pedagogy. In contrast, some other countries focus on pedagogy and are less demanding with regard to content (especially at the primary level). Yet other countries provide substantial mathematics and pedagogy training and have a long apprenticeship before teachers become certified.

These TEDS-M results (along with others) are quite important in framing the FIRSTMATH study that is detailed in this book. If beginning teachers at the end of their preparation have strong mathematics knowledge but rudimentary notions of pedagogy, where and when do they learn to teach? If beginning teachers at the end of their preparation have strong pedagogical preparation but less extensive knowledge of mathematics, where and when do they have opportunities to learn more mathematics? In both cases, how systematic are the opportunities provided? To what extent do teachers' beliefs or other characteristics influence their learning of mathematics or pedagogy during beginning teaching? These are just a few of the many questions that TEDS-M suggests and that FIRSTMATH takes up as worthwhile pursuits in a study of beginning teaching.

Beginning mathematics teaching is a strategic site for cross-national investigation Several years ago, I had a fascinating conversation with a teacher in Sweden about how she and her school colleagues met for a day near the end of a school year to prepare for the arrival of a new teacher in the next school year. She said that they first discussed what grade would be the best one to assign to a new teacher in the school. They observed that first grade was challenging because of all the group socialization issues children faced in that year. They also thought that third grade and beyond were challenging because of the academic expectations for the children and the widening range of proficiency across students in a class as the grade level increased. Thus, they decided that the new teacher should teach second grade.

After reaching that decision, they went through the class rosters of all the first-grade classes and identified all the children who were struggling to meet academic or behavior expectations. These “problem” children were assigned to more

experienced teachers who would also be teaching second grade in the coming year, and the new teacher's class roster was created to include only non-problematic students. They also discussed several additional aspects of the new teacher's assignment.

When I asked my teacher informant why she and her colleagues gave so much time and attention to the pending arrival of the new teacher, she replied that they wanted to be sure that the new teacher would be successful in her first year so that she would want to continue teaching and working at their school. I do not claim that this is typical of teachers and schools in Sweden, because I do not know if that is the case, but the anecdote reveals a few of the many complexities and policy/practice study opportunities that are associated with beginning teachers and teaching. Given how many teachers leave the classroom after only 1 or 2 years, the careful study of this transition from professional preparation to early professional practice is critical.

Toward a Study of Beginning Teachers and Teaching: FIRSTMATH Proof of Concept

This book presents FIRSTMATH—an exploratory pilot study of the knowledge, beliefs, and experiences of nearly 400 mathematics teachers in their early years of teaching in schools in 12 countries. The study addressed questions suggested by but not answered in TEDS-M. For example, FIRSTMATH demonstrates the feasibility of studying beginning teaching across widely varying contexts globally. It is obviously challenging to define beginning teaching in a way that would be consistent across countries and to obtain an appropriate sample to allow for comparative analyses. Moreover, FIRSTMATH tackled the design of instruments that validly and reliably measure the knowledge, beliefs, and experiences of mathematics teachers across countries in their early years of teaching. These matters and a host of other vexing technical challenges are addressed in the work reported in this book. In addition, the FIRSTMATH pilot study examined the added contribution of systematic opportunities to learn (e.g., in-service or induction programs) and on-the-job experience to current teacher preparation programs.

Maria Teresa Tatto and her FIRSTMATH colleagues have produced a book that will serve as a valuable resource to researchers who focus on teacher education in general or on mathematics education in particular. This book is especially important for those who wish to understand the conceptual, logistical, and methodological challenges entailed in the development of a scientifically rigorous study of teacher education and beginning mathematics teaching within and across countries. The authors' detailed explication of the problem space will encourage and support others to pursue this line of inquiry, and their careful attention to matters of technical quality will be valuable in providing instrumentation that can be used, adapted, and improved in further investigations, including taking FIRSTMATH to scale. The findings of a rigorous full-scale investigation built upon the extensive foundational work reported in this book will greatly benefit educational researchers, teacher

educators, and supporters of beginning teachers in their work. Perhaps even more importantly, the findings of such an investigation will also illuminate the nuances of the embedded issues and help policy-makers and other education decision-makers to resist overly simplistic characterizations and overgeneralized attributions that may not apply in all settings in which students acquire high levels of mathematics proficiency.

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Preface

The objective of this book is to share with the education community the process of developing and conducting a cross-national proof-of-concept (PoC) study of early-career mathematics teaching. The study is known as FIRSTMATH (First Five Years of Mathematics Teaching) and was funded by the US National Science Foundation with the goal of investigating beginning teachers' development of mathematical knowledge for teaching and the influence of previous preparation, school context, and opportunities to learn on the job, on that knowledge, on their practice, and on their pupils' learning.

Although answers to such essential questions are highly relevant, the key concern for funders and researchers at the time the proposal was funded was whether we could prove that such a study could be carried out with an acceptable level of rigor on an international scale, including the viability of the research design, methods, and instrumentation. Accordingly, the funding was limited, supporting a small study whose objective was to test the concepts, methods, and strategies for a larger comparative international research project. In addition, while we provide much detail in the book to describe what we have done, the actual instruments, methods, and strategies will be made available once the main study is underway to those who engage in the collaborative research endeavor we outline here. Once the study is completed, the instruments and data will be made publicly available.

The FIRSTMATH study sought to further the work done in the Teacher Education and Development Study in Mathematics (TEDS-M). Whereas TEDS-M investigated the influence of teacher education on the knowledge theorized as needed for teaching mathematics, the goal of FIRSTMATH was to develop rigorous methodologies to help uncover whether there are similarities or patterns in the ways that mathematics teachers develop within and across contexts.

The fundamental and larger question that the FIRSTMATH and TEDS-M studies sought to answer had to do with the development of theory: *Could a valid theoretical framework be developed to guide our study within and across a wide variety of countries and cultures? Is it possible to develop methods and instruments to help us test such theory and to understand how teachers learn to teach mathematics effectively to a wide variety of pupils across the primary and secondary grades? Can*

such a study help to explore the connections between preservice preparation and what is learned on the job as it concerns knowledge, skills, and curricular content and the degree to which standards, accountability, and other similar mechanisms operate to regulate the support that early-career mathematics teachers receive during their first years of teaching?

Consistent with calls to share scholarship in teaching and teacher education, in this book, we candidly report on the methodological challenges in implementing the FIRSTMATH PoC study as well as the progress made: development of a sampling frame, of instruments, innovative analysis methods, and more important the collaboration of an international network of researchers. While PoC studies typically do not report results, as the scope is limited, the chapters in this book indicate for the small samples who responded to our instruments the challenges that early-career teachers of mathematics encounter across the many dimensions we measured. Although limited in scope, these findings warrant an in-depth larger study.

Undertaking a large international study as the one we envision will be challenging but doable by learning communities of teachers and teacher educators. The analysis of the measures we developed provides evidence that the questions we asked are relevant and valid and should be pursued nationally and cross-nationally, especially because our study provides much needed indicators, statistical approaches, and monitoring tools to better assess the essential contribution that high-quality teaching and teacher preparation can play to ensure inclusive and equitable quality education for all children, which is the main drive of UNESCO's [Sustainable Development Goal 4](#): to “ensure inclusive and equitable quality education and promote lifelong learning opportunities for all” to be achieved in 2030.

An important contribution to scholarship is sharing not only the final results of successful studies but also the backstory of how these are built. Whereas in other fields these exchanges provide valuable learning experiences, we typically lack this kind of exchange in education. Thus, we offer this book in the hope that it will help the field move forward in the development of theory and a sustainable research agenda in education to improve mathematics teaching and learning for all.

Tempe, AZ, USA
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Maria Teresa Tatto

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Acronyms and Abbreviations

AERA	American Educational Research Association
APA	American Psychological Association
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CMC	Complex multiple choice
CR	Constructed Response
CSEM	Conditional Standard Error of Measurement
DIF	Differential Item Functioning
GDP	Gross Domestic Product
IEA	International Association for the Evaluation of Educational Achievement
IRB	Institutional Review Board
IRC	International Research Center
IRT	Item Response Theory
ISCED	International Standard Classification of Education
MC	Multiple Choice
MCK	Mathematical Content Knowledge
MNSQ	Mean Square
MPCK	Mathematical Pedagogical Content Knowledge
MT21	Mathematics Teaching for the Twenty-First Century
NCES	National Center for Education Statistics
NCME	National Council on Measurement in Education
NCRTL	National Center for Research on Teacher Learning
NRC	National Research Center
NSF	National Science Foundation
OECD	Organisation for Economic Co-operation and Development
OTL	Opportunity to Learn
P-TEDS	Pre-teacher Education Development Study
PISA	Programme for International Student Assessment
PoC	Proof of Concept
PRT	Pupil Response Theory

RMSEA	Root Mean Square Error of Approximation
TALIS	Teaching and Learning International Survey
TEDS-M	Teacher Education and Development Study in Mathematics
TFA	Teach for America
TF	Teach First
TIMSS	Trends in International Mathematics and Science Study
TLI	Tucker Lewis Index
UK	United Kingdom
UNESCO	United Nations Educational, Scientific and Cultural Organization
US	United States
VAM	Value-Added Measures

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