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Jorge Angeles · Damiano Pasini

Fundamentals of Geometry Construction

The Math Behind the CAD

Springer

Jorge Angeles
Department of Mechanical Engineering
McGill University
Montreal, QC, Canada

Damiano Pasini
Department of Mechanical Engineering
McGill University
Montreal, QC, Canada

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Beware of designers who walk around without paper and pencil.

Ancient Chinese proverb.

Preface

The purpose of the book, in a nutshell, is to provide both the beginner and the experienced CAD user with *the math behind the CAD*. It is expected that, with the geometry tools introduced here, the reader will be able to exploit CAD software to its fullest extent. In fact, the book should allow the reader to go *beyond what CAD software packages offer in their menus*.

Chapter 1 includes a summary of facts from basic Linear and Vector Algebra, most of which the reader may have covered either in high school or in undergraduate math courses. The chapter also includes some new items, such as the 2D form of the vector product. In fact, while the vector product is defined solely in 3D, it is needed in some 2D applications, as made apparent in this chapter. The 2D form is not common in mathematics textbooks, although it is a useful tool that helps to streamline many calculations, as the reader will find here. One more novelty, not covered in either high school or undergraduate math courses, is the manipulation of block matrices and vectors. This material is introduced to help the reader compute inverses and determinants of $n \times n$ matrices, for $n > 3$ using the corresponding expressions for 2×2 and 3×3 matrices. These calculations become handy in Chap. 4, devoted to affine transformations.

Chapter 2 includes the study of the relations among points, lines, and curves in the plane. Here, the difference between curves representing functions and their geometric counterparts is emphasized. Plotting the former is quite a different task from plotting the latter. Scientific software makes the difference, in that it uses different commands to plot one or the other. The former are representable by means of *explicit functions*, e.g., $y = f(x)$, the latter by *implicit functions*, e.g., $f(x, y) = 0$. Geometric concepts are studied in order of growing complexity, from points to lines, then algebraic curves and, finally, non-algebraic and free-form curves. The simplest algebraic curves, conics, are given due attention. However, these are too limited for practical engineering design applications. For example, they cannot produce a smooth fillet to join the two lines of a corner with tangent and curvature continuity, which is essential to avoid stress concentrations in mechanical design. For this reason, Lamé curves are introduced in this chapter.

Chapter 3 is devoted to geometric objects in 3D, namely, points, planes, lines, and surfaces. Of the latter, only quadrics are studied, to keep the discussion at an elementary level. It is shown that, with these simple surfaces and planes, more complex surfaces can be generated by means of *Boolean operations*. These are at the basis of CAD software. Furthermore, as illustrated in Chap. 4, highly complex surfaces, like those of screws, can be readily generated by means of affine transformations.

The concept of *affine transformations* is indeed a key component of Chap. 4, which includes applications of these transformations to the synthesis of curves and surfaces that could be extremely cumbersome to produce otherwise. Again, affine transformations lie at the core of CAD software. Understanding these transformations should help the reader best use the software tools available in the market.

In this book, catering to various disciplines—engineering, graphic design, animation, and architecture—an attempt is made to keep the material discipline-independent, while including some examples of interest to various disciplines. Furthermore, the book can be used to complement lectures on CAD, at the undergraduate level. In this light, the references included in footnotes are not mandatory reading. They are included for the benefit of both the lecturer and the curious, diligent student wanting to go deeply in certain subjects.

A crucial point that led to the production of the Lecture Notes, and hence to this book, was the input received from Axel Pavillet, Ph.D., alumnus of the French *Ecole Polytechnique*.[1,2] In the early 2000s, Dr. Pavillet was an Instructor of the course on *engineering drawing* offered by the Department of Mechanical Engineering at McGill University. The syllabus of this course included, as was usual in those days, a substantial component of *Descriptive Geometry* (DG). Dr. Pavillet's point was that DG was completely out of place since the end of the twentieth century, given that computer-graphics software was already available in the 80s. Lengthy exchanges with Dr. Pavillet and other instructors finally led us to the idea that the course in question needed an in-depth revision, with classical DG replaced with fundamental concepts of geometry and linear algebra. Thus came this book to fruition.

This book is the result of teamwork. We thank all the students, undergraduate and graduate, who contributed with their work to bring the book to its current form. We acknowledge especially the work of two Ph.D. students who were instrumental in both the first phases of the book editing and producing graphic material—photos and drawings—that led to the final version. The highly professional work done by Vikram Chopra and Wei Li is dutifully acknowledged, with our most sincere thanks. The editing work leading to the final version was conducted by Dr. Bruno Belzile, Postdoctoral Fellow at the *Robotic Mechanical Systems Laboratory* (RMSLab). The RMSLab is affiliated with both the *Centre for Intelligent Machines* and the *Department of Mechanical Engineering*, McGill University, in Montreal. Dr. Belzile provided also technical support in the graphics work that made possible

[1]Asancheyev (2002).

[2]Pavillet (2011)

the completion of this highly demanding project. Our most sincere acknowledgement to Dr. Belzile for his diligent and excellent work.

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Any suggestions for improving or any reports of typos or inaccuracies are most welcome.

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Jorge Angeles
angeles@cim.mcgill.ca

Damiano Pasini
damiano.pasini@mcgill.ca

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